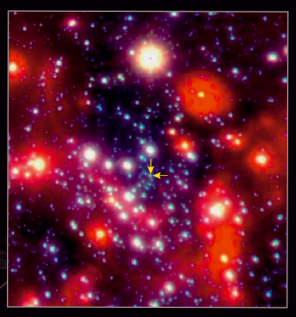
THE ESA/ESO ASTRONOMY EXERCISE SERIES

Student exercises in astronomy using observations from the NASA/ESA Hubble Space Telescope and the ESO telescopes



Exercise 6

The mass of the black hole at the centre of our Milky Way

Based on Observations with the ESO Very Large Telescope

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The ESA/ESO Astronomy Exercise Series 6

The mass of the black hole at the centre of our Milky Way

Astronomy is an accessible and visual science, making it ideal for educational purposes. Over the last few years the NASA/ESA Hubble Space Telescope and the ESO telescopes at the La Silla and Paranal Observatories in Chile have presented ever deeper and more spectacular views of the Universe. However, Hubble and the ESO telescopes have not just provided stunning new images, they are also invaluable tools for astronomers. The telescopes have excellent spatial/angular resolution (image sharpness) and allow astronomers to peer further out into the Universe than ever before and answer long-standing unsolved questions.
The analysis of such observations, while often highly sophisticated in detail, is at times sufficiently simple in principle to give secondary-level students the opportunity to repeat it for themselves.

This series of exercises has been produced by the European partner in the Hubble project, ESA (the European Space Agency) – which has access to 15% of the observing time with Hubble – together with ESO (the European Southern Observatory).



Figure 1: The ESO Very Large Telescope
The ESO Very Large Telescope (VLT) at the Paranal Observatory (Atacama, Chile) is the world's largest and most advanced optical telescope. With its supreme optical resolution and unsurpassed surface area, the VLT produces very sharp images and can record



Introduction

Black holes, an introduction

Black Holes may seem mysterious, but they consist of the same ordinary matter that makes up the Sun, the Earth and everything on it. The main difference is that the matter in a black hole is squeezed into an incredibly small volume. If the Earth were to become a black hole, it would have to be compressed to the size of a marble – about 1 centimetre in radius. Newton's law of gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

tells us that the attractive force F between two masses m_1 and m_2 increases as the square of the distance r between the two bodies diminishes. Here on the Earth's surface we are about 6378 km from the Earth's centre. On a marble-sized Earth we would only be 0.5 cm from its centre. This huge reduction in r makes the gravitational attraction more than a billion times greater than on Earth normally.

It is this large force that allows a lot of strange things to happen to anything that gets too close to a black hole. For example, there is a point of no return, called "the event horizon". Once inside, nothing, not even light, can escape. Furthermore, the strong gravitational attraction close to the black hole means that anything moving around it must travel at enormous speeds to avoid spiralling into the hole. If these high velocity pieces of material collide with each other, the collision is disastrous and produces large amounts of heat and light. In this exercise we will learn more about black holes.

How black holes got their name

The scientist John Wheeler coined the name 'black hole' in 1967. It was termed a 'hole' because things that pass the event horizon will never re-emerge. In fact, precisely nothing can escape a black hole. Objects can escape from the Earth if they are shot away with speeds larger than 11 km/s. This is a tremendous velocity. But to escape a black hole, an object would need a velocity greater than the speed of light - about 300000 km/s! However, according to the theory of relativity, nothing in Nature can move faster than the speed of light. In other words, not even light escapes: it is truly a 'black' hole.

So, things disappear inside the black hole never to reappear.

Originally, many scientists considered black holes to be just a nice idea on paper, not something that really existed. Today we have very strong evidence that there is a black hole right at the centre of our own galaxy, the Milky Way. In this exercise we will re-discover this black hole and determine its mass.

The black hole at the centre of our Milky Way

The first hint that there might be a black hole lurking at the centre of the Milky Way came when people noticed a highly unusual source of radio emission in the southern constellation of Sagittarius (see Figure 2). This source was named "Sagittarius A*" (SgrA*). It was clear that the unknown source of the radio emission could not possibly be a star and it was speculated that the mystery source might be a black hole at the centre of the Milky Way. Matter circling a black hole at high speed could account for the unusual radio emission signal.

Unfortunately, a black hole is extremely small and completely black so we cannot hope to see it directly. Evidence for a black hole can be obtained by measuring two quantities near the suspected black hole:

(1) the speed of the material orbiting it and

(2) the light coming from the area.

The speed tells us about the mass concentrated in that volume of space while the light emitted tells us if this mass could be in the form of stars. There are a lot of stars in motion at the centre of the Milky Way. In this part of the exercise we will use real observations from the centre of the Milky Way to find those stars and measure their speeds.



Introduction

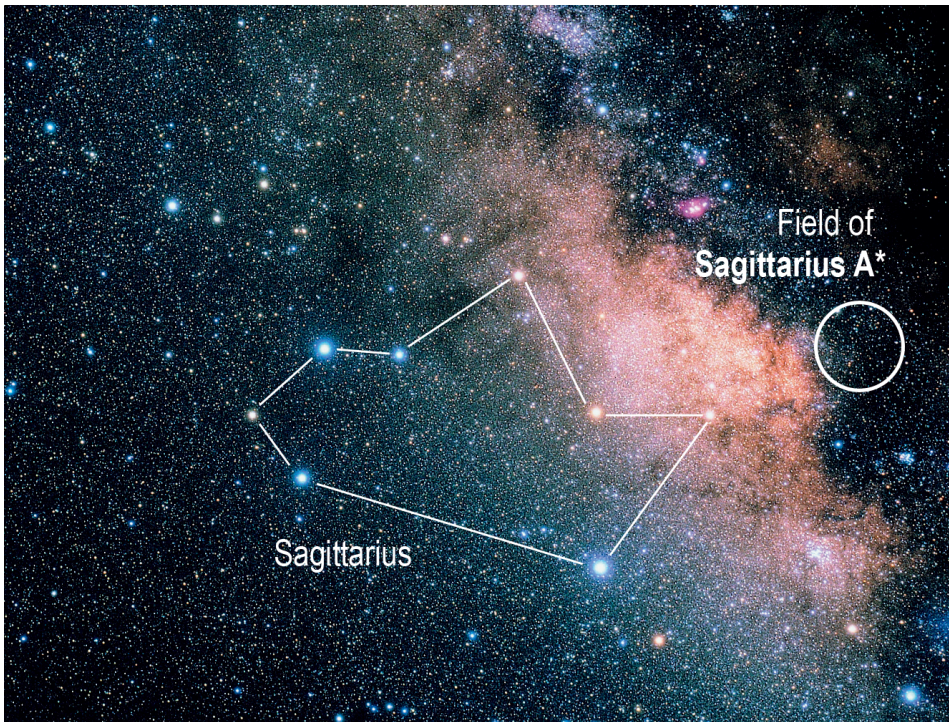


Figure 2: The "teapot" asterism in the constellation of Sagittarius and the field of Sagittarius A*. *You see a photo of the night sky around the "teapot" part of Sagittarius. Sagittarius can best be seen from the Southern hemisphere. The radio source Sagittarius A* is located in the center of the white circle.*

Gravitation

In the early 1600s, Johannes Kepler deduced the three laws that bear his name and describe the way planets move around the Sun:

1. Planets move in elliptical orbits around the Sun. The Sun is at one focal point of the ellipse.

2. The area A crossed by the line joining the Sun and the moving planet per unit time is a constant value:

$$\frac{A}{\Delta t} = \text{constant}$$

3. The square of the period P of the orbit of the planet is proportional to the cube of the semi major axis of the an elliptical orbit (which is half the distance of the longest axis of an ellipse). It was later shown that P can be computed from:

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$$

where G is the gravitational constant and m_1– the mass of the Sun and m_2 – the mass of the planet.

Task 1 (optional task) – Ellipses

The first law states that planets move in ellipses. An ellipse with centre (0,0) is the curve that goes through the points (x,y) which have the relation:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \qquad \textbf{(1)}$$

Take a = 10 and b = 5 and compute various points (x,y) using formula **(1)**. Connect the points to draw an ellipse. Repeat this, but now for a = 10 and b = 2.

? Which features do the values a and b correspond to in the ellipse? Finally, what kind of figure does the ellipse become when a = 10 and b = 10?

Figure 3 shows an ellipse. The lines of length a and b are called the semi-major axis and semi-minor axis, respectively. An ellipse has two focal points. The total distance from one focal point through any point on the ellipse to the other focal point is a constant 2a. This fact is used by gardeners to make flowerbeds with an elliptical shape. Put two pins on a sheet of paper. Connect the pins with a loose loop of thread. Now put a pencil on the paper inside the loop and move it around while keeping the thread stretched. An ellipse will appear with focal points at the location of the pins. As the distance between the pins decreases the ellipse will become rounder, and eventually form a circle in the extreme case when the distance between the pins is reduced to zero and they are placed at the same point.

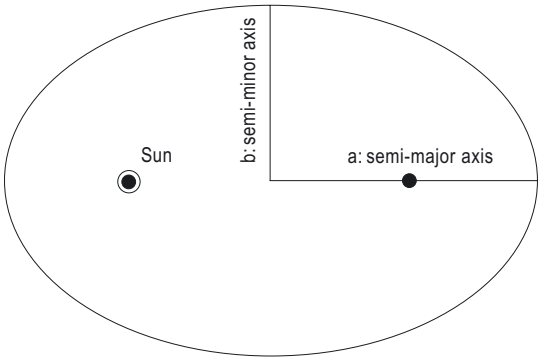


Figure 3: *An ellipse, where the semi-major and minor axes and the focal points (black dots) are shown. Planets orbit in ellipses around the Sun, which is in one focal point, as indicated.*

The second law is illustrated in Figure 4.

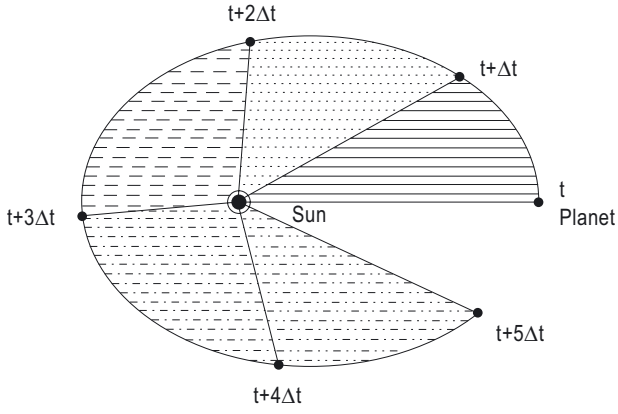


Figure 4: *Kepler's second law says that a planet (the small dot on the ellipse) moves around the Sun (indicated by the encircled dot) in such a way that in equal amounts of time Δt the line joining planet and Sun traverses equal amounts of area. The equal areas each have their own shading. The orbital velocity is higher when the planet is close to the Sun.*

The orbital period of a planet is the time it takes

it to make one full ellipse around the Sun. Kepler's third law says that if you know two of the following three quantities: the period (P), the semi-major axis (a) and the total mass ($m_1 + m_2$) of Sun and planet together, you can compute the unknown one.

Task 2 (optional task)
The mass of Sun and Earth

? Compute the mass of the Sun and Earth together, using Kepler's Third Law and the fact that the semi-major axis of the Earth's orbit is 150 million km and its period is 1 year and $G = 6.67 \cdot 10^{-11}\ m^3\ s^{-2}\ kg^{-1}$

Kepler published his laws in 1609 and 1619. Some years later, in 1687, Isaac Newton showed that these laws are a consequence of the universal law of gravitation. This means that gravitational forces act on any two bodies with mass, not just the Sun and a planet. In other words, gravitation makes a stone fall to the Earth; it keeps the Moon in its orbit; the Earth in orbit around the Sun; and the Sun in orbit around the centre of the Milky Way or any star in orbit around a black hole. From the 1990's on, astronomers have found stars moving very fast at the centre of the Milky Way. Could these high speeds be caused by a massive black hole at the centre of our Milky Way?

The observations

Observations of stars near the centre of the Milky Way are difficult. The many stars and dusty clouds between us and the centre obscure our view out towards the centre. Fortunately, infrared light has a longer wavelength than visible light and is much less obscured by the dusty clouds so infrared light from stars at the centre can reach us. A team of astronomers, led by German astronomer Reinhard Genzel, has made images at infrared wavelengths of the centre of the Milky Way using the ESO Very Large Telescope in Chile (see Figure 5) over many years.

In successive images, taken at different times, the stars near the Milky Way centre move a bit. One star in particular, called S2, has moved a lot over the years. Its position when it was near the centre of the Milky Way is shown in Figure 6.

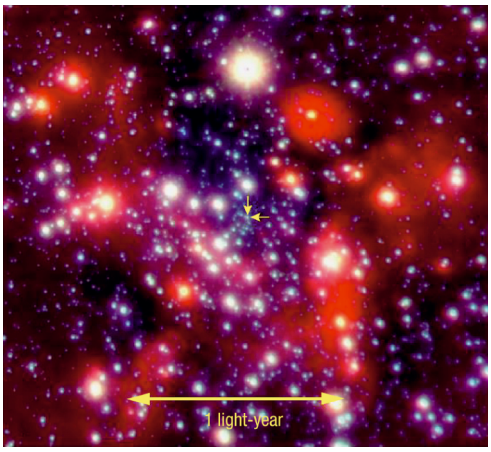


Figure 5: *a near-infrared image of the few central light-years of the Milky Way obtained with the NACO instrument at the Very Large Telescope. The compact objects are stars: blue stars are hot and red stars are cool. The two yellow arrows mark the position of the black hole candidate "Sagittarius A*" at the Milky Way centre. The yellow bar indicates the length of 1 light-year at the centre, which corresponds to an angular distance of eight arcseconds.*

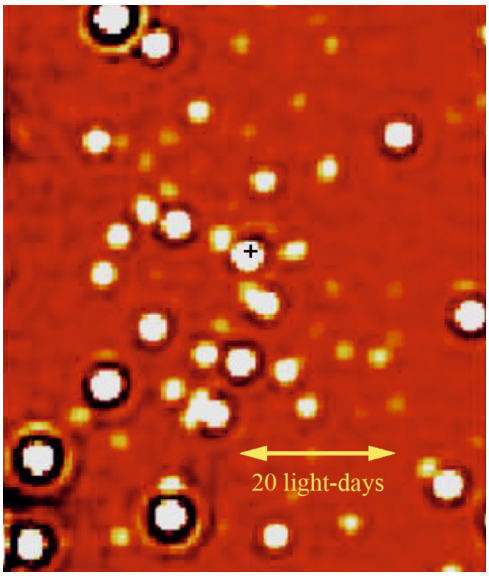


Figure 6: *A near-infrared image of the central ~2 arc-seconds of the Milky centre corresponding to ~82 light-days. The radio source SgrA* at the Milky Way centre is indicated by the cross. The white dot at almost the same point is the star S2.*

Date (year)	x (arcsec)	dx (arcsec)	y (arcsec)	dy (arcsec)
1992.226	0.104	0.003	-0.166	0.004
1994.321	0.097	0.003	-0.189	0.004
1995.531	0.087	0.002	-0.192	0.003
1996.256	0.075	0.007	-0.197	0.010
1996.428	0.077	0.002	-0.193	0.003
1997.543	0.052	0.004	-0.183	0.006
1998.365	0.036	0.001	-0.167	0.002
1999.465	0.022	0.004	-0.156	0.006
2000.474	-0.000	0.002	-0.103	0.003
2000.523	-0.013	0.003	-0.113	0.004
2001.502	-0.026	0.002	-0.068	0.003
2002.252	-0.013	0.005	0.003	0.007
2002.334	-0.007	0.003	0.016	0.004
2002.408	0.009	0.003	0.023	0.005
2002.575	0.032	0.002	0.016	0.003
2002.650	0.037	0.002	0.009	0.003
2003.214	0.072	0.001	-0.024	0.002
2003.353	0.077	0.002	-0.030	0.002
2003.454	0.081	0.002	-0.036	0.002

Table 1: ***Coordinates of star S2***
Column 1: date on which the position of star S2 was measured (e.g., 2000.500 means exactly in the middle of 2000)
Column 2-5: x and y position of the star and the uncertainty on both coordinates. The units are in arcseconds. The putative black hole is located at (0.0, 0.0).

The mass computation

With the positions of S2 listed in Table 1 we can determine the mass in SgrA* using Kepler's laws. Masses are mentioned in Kepler's third law. So we can use that law to find the mass of SgrA*. The law states that if you want to find the total mass $m = m_{BH} + m_S$, i.e. the mass of the black hole (m_{BH}) and the star (m_S) together, we need to know the period (P) and the semi-major axis (a) of the stellar orbit.

You will first find out the total mass and then later figure out how much belongs to the black hole and how much to the star.

Task 3 – Finding a

You can determine the semi-major axis (a) of the stellar orbit by fitting an ellipse to the positions of S2 as listed in Table 1.

? Plot all positions on a piece of measurement paper to scale or use a plotting routine from a computer program.

? Indicate the uncertainty on the x and y position for each point. You can do this by drawing bars with the size of the uncertainty.

? Draw an ellipse by eye that best matches these measurements. The ellipse does not have to go through the points exactly because of the uncertainties in the positions.

? Now measure the semi-major axis in arcseconds. Convert this to a length in lightdays using the fact that 2 arc-seconds correspond to 82 lightdays at the Milky Way Centre.

? Compare your measurement to the results of others in your class and use this information to estimate the uncertainty of your measurement. The semi-major axis of the ellipse that best fits the measurements can also be computed more precisely using a mathematical formula. Your teacher has the result of such a computation. If the difference between your measurement and the mathematical computation is larger than your uncertainty, you probably have underestimated the uncertainty on your measurement. If the difference between the two values is much smaller than your uncertainty, you might have overestimated the uncertainty.

? Now you need to find the period (P) of the orbit of S2. In the time the star goes around the black hole once the con-

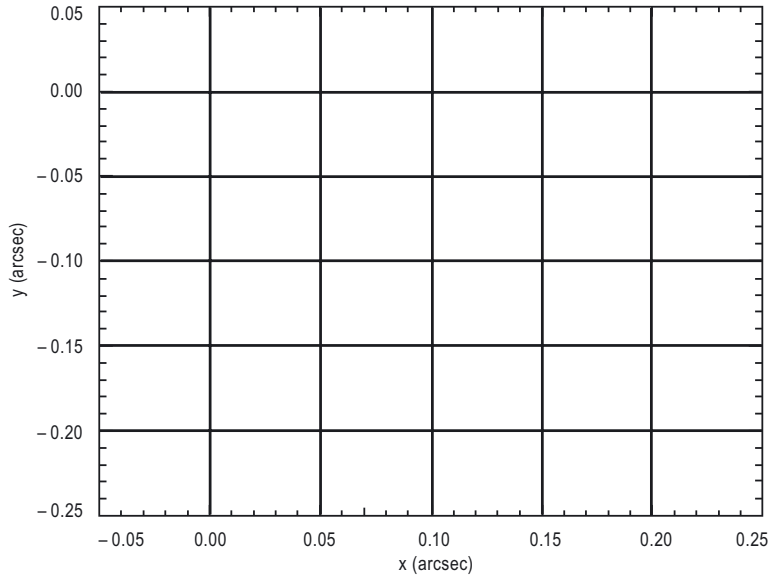


Figure 7: *Empty plot for task 3.*

nector – the line between the black hole and the star – maps out the area of the ellipse. The area (A_{ell}) of an ellipse is:

$A_{ell} = \pi \times a \times b$

Kepler's 2nd law tells you that the area traversed by the connector is proportional to the time spent in this area.

For example, in half the period, i.e. P/2, the connector will map out half the area = A_{ell} /2. More generally in the time Δt it takes the star to get from position 1 to 2, the connector traverses an area:

$$\Delta A = \frac{\Delta t}{P} A_{ell} \qquad (2)$$

To get P from this formula, you thus need to determine ΔA, Δt and A_{ell}.

Task 4 – Finding P

Choose one of the two methods below for this exercise.

Method A: using paper and a scale
If you are reluctant to count the number of squares in the ellipse, you can use this method of finding areas. Cut out the ellipse using scissors. Weigh the cut out ellipse on a scale that has an accuracy of 0.01 g. This will give you a value for A_{ell}.

Cut the area that was not crossed by the line between the black hole and S2 away, and weigh the segment. This will give you a value for ΔA. If you wanted the actual areas, the masses of the paper pieces should be converted into square arc-seconds. But since you only need the ratio of the areas ΔA and A_{ell}, there is no reason to do this conversion.

The accuracy of this method can be improved by gluing the paper to a piece of cardboard before cutting (be sure to distribute the glue evenly!).

? Compute P from formula (2) using your measurements of the mass of the segments of the ellipse. The value of Δt can be determined from the dates of the measurement given in Table 1. Compare this with the result given by your teacher.

Method B: using paper and dividing the work

? Measure A_{ell} and the ΔA between each two positions from the drawing you made in exercise 3. Divide this work over the class. You can estimate areas by counting the squares on the measurement paper. Now compute P from formula (2) for each two positions. Compare this with the result given by your teacher.

Task 5 – Finding the total mass m

? Now compute the total mass m of star and black hole together using Kepler's third law.

Is there really a black hole?

How much of this mass belongs to the star and how much belongs to the black hole?

Stars have masses which range from 0.08 to ~120 Solar masses. The total mass computed in task 5 is much larger. Thus, regardless of what kind of star we are dealing with, it has a negligible mass m_S compared to the mass of the black hole m_{BH} (i.e., $m_{BH} >> m_S$) . Thus almost all the mass in the total mass m comes from the black hole. Wait a minute…, do you actually know that this mass belongs to a black hole? You have only computed a mass. Perhaps this mass does not belong to a black hole, although it is certainly equivalent to a large number of stars. The difference between these two possibilities is that stars emit light and a black hole does not.

? How much light would you expect from SgrA* if all this mass were in the form of stars?

Task 6 – A black hole or many stars?

To a first approximation you can assume that all the mass belongs to stars just like the Sun. The mass of the Sun – the 'solar mass' – is about 2×10^{30} kg.

? How many suns (N_{sun}) would you need at the location of SgrA* to account for the mass?

The luminosity of the Sun is about 4×10^{26} W.

? What is the luminosity you would expect from the measured mass under your assumption of many 'suns'?

Astronomers often use 'magnitudes' instead of luminosities. Please read pages 5 and 6 of the Astronomical Toolkit to learn about apparent and absolute magnitudes. Read page 5 on how convert between these two magnitudes using the distance equation. The absolute magnitude (M) of the Sun is M = +4.83. The distance (D) to the centre of the Milky Way is about D = 8.0 kpc.

? Compute the apparent magnitude of the Sun if it were at the Milky Way centre. What would be the apparent magnitude if there were N_{sun} Suns at the location of SgrA*?

Astronomers have measured almost no light coming from the Milky Way centre. This can also be seen from Figures 4 and 5. They show that the light coming from the location of SgrA* is less than that of the surrounding stars. The conclusion is that the Milky Way centre is much too dark to allow stars to account for the measured mass. So this mass must be due to a black hole.

Small and large black holes

You might wonder if black holes have to be as massive as the one you just found at the centre of the Milky Way. In the introduction the definition of a black hole is given as an object from which light cannot escape. Let us have a look at escape speeds for different objects.
The escape velocity from a spherical body with mass (m) and radius (r) can be given as:

$$V_{esc} = \sqrt{\frac{2Gm}{r}} \qquad (3)$$

Task 7 – What makes a black hole?

? Compute the escape velocity from the Earth using the Earth's mass $m_{Earth} = 6 \times 10^{24}$ kg and its radius $R_{Earth} = 6378$ km.

Now compute the escape velocity if the radius of the Earth were only 0.5 cm.
Finally, compute the escape velocity if the Earth had its usual radius but had a mass that were 2200 times that of the Sun.

You see that the Earth is transformed into a black hole in two cases: if you compress it enormously or if you add an enormous amount of mass to it. The Sun has a radius that is a little over hundred times that of the Earth. Therefore, the second case means that you are squeezing an object with 2200 times the mass of the Sun into an object with a radius more than 100 times smaller than the Sun, in other words, also an enormous compression. The crucial property that makes a black hole a black hole is not mass or radius, but 'compactness'. This is the ratio of mass to radius, and equation (3) shows this in mathematical terms.

Task 8 (optional task) If you were a black hole?

Does the compactness mean that you could have black hole masses much smaller than the Earth provided that you make them very small?

? Take the mass of your own body and compute the radius needed to get an escape velocity equal to the speed of light. This is the radius you would have to squeeze your self in order to transform yourself into a black hole. Compare this radius to the typical size of one atom: 2×10^{-10} m.

Conclusion

You see that anything, the Sun, the Earth and even you could be a black hole if you were able to make it compact enough. However, in the Universe we have only found black holes with masses larger than a solar mass so far. Sometimes they are much larger, as for the one you found in SgrA* at the centre of the Milky Way. This is not too unexpected. You yourself do not collapse into a black hole spontaneously. It would require a kind of 'compression-machine' that neither mankind nor nature has. The same holds for the Earth. It will not collapse by itself. Matter is sturdy enough that the inner layers of the Earth can carry the layers on top of them. It is only for stars much heavier than the Sun, that matter cannot carry the layers on top of it

indefinitely. Initially, such a star resists its collapse by burning fuel at its core that produces light and heat just like the Sun. The heat and pressure help the inner layers support the outer layers. However, at some point the star runs out of fuel and stops shining. At this point the inner layers cannot resist the pressure from the outer layers anymore and the star collapses into a black hole – astronomers call such an event a 'supernova'.

The universal tale thus comes to a conclusion. If you are too small in this Universe you run no risk of collapse, but…you do not shine. If you are too large, you will shine brightly in life but end burned-out and black. If you can find a balance between these two extremes, like our Sun, you can combine a bright life with a gentle end.

References

Articles on which exercise is based

- Schödel, R., Ott, T., Genzel, R., Eckart, A., Mouawad, N., & Alexander, T. 2003, Astrophysical Journal, 596, 1015
- Schödel, R., et al., 2002, Nature, 419, 694

A few starting points on the web

- http://www.eso.org/outreach/press-rel/pr-2002/pr-17-02.html
- http://amazing-space.stsci.edu/capture/blackholes/
- http://hubblesite.org/discoveries/black_holes/
- http://www.phys.vt.edu/~jhs/faq/blackholes.html

Books

- Igor Novikov, "Black Holes and the Universe", Cambridge University Press, Cambridge
- Stephen Hawking, "A Brief History of Time", Bantam Books, Toronto, New York
- Mitch Begelman & Martin Rees, "Gravity's Fatal Attraction : Black Holes in the Universe", Scientific American Library Series, W. H. Freeman
- Kip Thorne, "Black Holes & Time Warps", Norton, New York, London

Colophon

The ESA/ESO Astronomy Exercise Series
Exercise 6: The mass of the black hole at the centre of our Milky Way

Produced by:
the Hubble European Space Agency Information Centre and the European Southern Observatory:
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Acknowledgements:
Thanks to Rainer Schoedel for providing the data to construct Table 1, and to Rosa M. Ros, Richard West and Douglas Pierce-Price for providing valuable inputs.

ESO

European Organisation for Astronomical Research in the Southern Hemisphere



Teacher's Guide

Introduction

Goal: The principal goal of this exercise is to determine the mass of the black hole at the centre of our own Milky Way. The underlying intent is to explain the basic properties of black holes and to counter some misconceptions about them.

Pre-requisite knowledge: A firm grasp of basic mathematics and some idea of ellipses and Kepler's Laws on orbits, although the latter two are discussed in the exercise.

Optional tasks: Tasks 1, 2 and 8 are indicated as optional because they are not essential for understanding basic black hole properties and estimating the mass of the black hole at the centre of the Milky Way.

Sources for material

This exercise is based mainly on the observations of S2 presented in the article Schödel et al. (2003) which is available on the website of the Astrophysical Journal (subscription needed). Alternatively, a similar version of the article can be obtained at http://xxx.lanl.gov/abs/astro-ph/0306214 (no subscription needed). As discussed in the exercise, to infer the mass of the black hole at the centre of the Milky Way one needs to determine the period and the semi-major axis of the stellar orbit. These measurements should be made in the plane of the orbiting star. The observed locations listed and shown in Schödel et al. (2003) are in the plane of the sky. However Table 1 in the exercise lists the coordinates in the plane of the orbit. These results were computed by rotating and deprojecting the coordinate system used in Schödel et al. (2003).

Solution of exercises

Task 1 (optional task)

Have the students isolate y on one side of the equation $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ and substitute values for x in the expression on the other side. They should use a = 10 and b = 5 and plot the result in the (x,y) plane. The resulting graph should be an ellipse with semi-major axis 10 and semi-minor axis 5. They should then repeat the plotting exercise with a = 10 and b = 2. This should make clear to them that a and b correspond to the semi-major and semi-minor axis respectively. Finally, they should repeat the plotting exercise using a = 10 and b = 10 to realize that a circle is a special kind of ellipse.

Task 2 (optional task)

Have the students use: $P^2 = \frac{4\pi^2}{G(m_1+m_2)}\ a^3$

Let them isolate $(m_1 + m_2)$ on one side of the equation. The variables a and P are given in the exercise. The result should be the combined mass of the Sun and Earth $\mathbf{(m_1 + m_2) = 2.0 \times 10^{30} kg}$.

Task 3

The students should plot all positions listed in Table 2 on a piece of graph paper to scale or use a plotting routine from a computer program. The plot should look similar to Figure 8 below.
NB: It is important to use graph paper if method B is to be used in exercise 4.

Students should also indicate the uncertainty on the x and y position for each point. Alternatively (to save time), the teacher can provide a graph with the positions. The students should draw the ellipse that best matches these measurements by eye. The ellipse does not have to go through the points

exactly because of the uncertainties in the positions. The students can measure the semi-major axis (possibly taking into account different scales of x and y axis). The students should convert the result from angular size to physical size using the fact that 2 arc-seconds corresponds to 82 light-days at this distance.

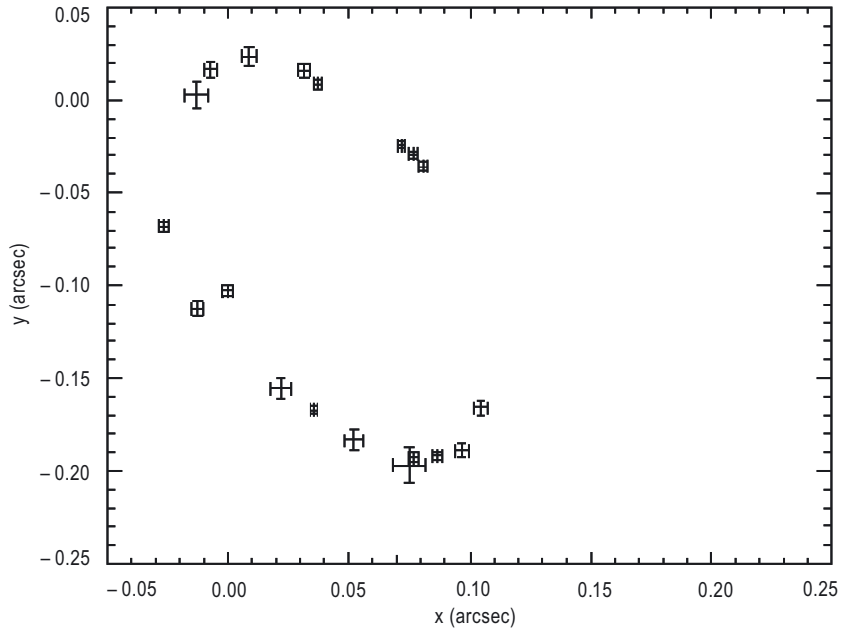


Figure 8: *Schematic view of the observed locations of S2 in the orbital plane.*

The precise value from a mathematical fit (see Schödel et al. 2003) gives a semi-major axis of the orbit of **S2 = 5.4 ± 0.4 light days**. The students should realize that they probably have under (or overestimated) their uncertainty if the difference between their result and the precise mathematical value is much larger (or smaller) than their estimated uncertainty.

Task 4

This problem can be solved by any of the two offered methods. See also the exercise itself for details for each method.

Method A: be sure to use an accurate enough scale.

Method B: from the plot in exercise 3 the students should draw lines from the origin to the 19 S2 positions. The area of each of the 18 orbital segments can be determined graphically by counting the squares. The precise period of the orbit of **S2 is 15.7 ± 0.7 yr**, given in Schödel et al. (2003).

Task 5

Have the students use: $P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$

Isolate $m_1 + m_2$. The variables a and P should have been derived by the students in exercise 3 and 4. The student should obtain a result of about 6.5×10^{36} kg ($\pm 2.0 \times 10^{36}$ kg) for $(m_1 + m_2)$, which is **$3.3 \times 10^6 \pm 1.0 \times 10^6$ solar masses.**



Task 6

The students should derive the number of suns N_{sun} from:
N_{sun} = [mass of black hole from task 5] / [mass of Sun (= 1.989 x 10^{30} kg)] = **3.3×10^{6}**
Following this they should compute the expected luminosity from these Suns by multiplying the given luminosity of the Sun by N_{sun}, i.e., $N_{sun} \times 4 \times 10^{26}$ W = **1.24×10^{33} W**

The students should now compute the apparent magnitude (m_{app}) of the Sun at the centre of the Milky Way using the distance modulus. Magnitudes and modulus are explained in the Astronomical Toolkit of the Astro-exercises. This will yield $m_{app} = M_{abs} + 5.\log(D(pc)) - 5 =$ **19.3**. The absolute magnitude of the Sun M_{abs} and the distance to the Milky Way centre D are given in the exercise.

Lastly, the students have to compute the apparent magnitude, $m_{app}(n_{sun})$, of N_{sun} Suns at the Milky Way centre. They can do this as $m_{app}(n_{sun}) = m_{app} - 2.5\log(N_{sun}) = 3.1$.

This formula can be derived from the definition of apparent magnitude given in the astronomical toolkit.

Task 7

The students should compute the escape velocity from the Earth using the formula given in the exercise: $V_{esc} = (2Gm/r) =$ **11.3 km/s**. The mass of the Earth (m) and the radius (r) are given in the exercise.

Then the students should recompute the escape velocity for Earth similarly, but using a radius = 0.5cm. This will yield an escape velocity = **4.0×10^{8} m/s**.

Lastly, the students can compute the escape velocity from Earth if its mass were to be 2.2×10^{3} $m_{sun} = 4.37580 \times 10^{33}$ kg. This yields an escape velocity = **3.0×10^{8} m/s**.

Task 8 (optional task)

The students should compute their own radius if they were a black hole by rewriting the formula for the escape velocity: $r = 2.0\ Gm/V_{esc}^{2}$. The student should then fill in their mass (m) and the speed of light (299792.458 km/s). For a body mass m = 60 kg, the result is **8.9×10^{-26} m**.

Feedback

Please do not hesitate to contact us: Hubble European Space Agency Information Centre (lars@eso.org) with specific questions or for more general information about the exercise. We would very much appreciate feedback on the exercise so that we can improve subsequent versions.

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