

Corrections to Exercise 4 (v4.0 17.04.02)

1. The stated “reasonable accuracy” of 20% on the distance (page 10) is an underestimate. If the calculations are done without extinction correction (see below) an accuracy of 50% will be a very good achievement. The students’ results for the distance will in this case be too high.

2. In order to get more ‘reasonable’ results we recommend to introduce the students to correction for “interstellar extinction” on the distance modulus $m-M$ (this is a simplification since there also is some – smaller – extinction influence on the B-V (or temperature) term). Interstellar extinction is caused by dust between us and the star, and will cause the observed magnitudes to be too large (the stars will be too faint) as well as a reddening of the observed colours (not corrected for here).

In summary we recommend using the equation:

$$D = 10^{(m-M-A+5)/5}$$

A is given by Harris et al. to 0.57 magnitudes (in the V band, which is what affects $m-M$).

Read more about interstellar extinction in:

<http://www.astro.virginia.edu/class/hawley/astr124/ism.html>

<http://tesla.phys.unm.edu/a111labs/cepheids/mags.html#Red>

The measurements quoted in the Teacher’s guide for Task 9-11 include the extinction correction, but due to a mistake the extinction concept is not included in the student’s part of the exercise.

Without extinction correction the error on the age can easily be a factor of a thousand or so (e.g. $15,000 \times 10^9$ years).

3.

- a) In the printed Teacher’s Guide Task 9-11 we read incorrectly the graph in Figure 3. The correct value for $m_V - M_V$ is 13.9. The corrected sections are found below.
- b) In the printed Teacher’s Guide Task 12-14 we, due to a mistake, used the Harris et al. value (4900 pc). To give a more realistic picture of the sort of accuracy to be expected for the calculations of the age we below repeat the calculations with our own distance measurement of 4.634 kpc.

We here repeat the full calculation of Task 12-14:

Task 12-14

A star at the turn-off point has an apparent magnitude of 18.7 in our measurements. Scientists have measured 18.3 (Rosenberg et al.).

Calculation of ration (I_{cl}/I_{Sun}):

As I_{Sun} is much larger than I_{cl} , the ratio will be a very small number, so we suggest calculating I_{Sun}/I_{cl} and then taking the reciprocal value for further calculation.

$$(I_{Sun}/I_{cl}) = 10^{(m_{cl} - m_{Sun})/2.5} = 10^{(18.7 - (-26.5)) / 2.5} = 10^{18.08} = 1.202 \times 10^{18}$$
$$\text{so } (I_{cl}/I_{Sun}) = 8.318 \times 10^{-19}$$

Further calculations:

$$(D_{cl}/D_{Sun}) = (4634 \times 3.086 \times 10^{13}) / 1.498 \times 10^8 = 9.559 \times 10^8$$

$$(L_{cl}/L_{Sun}) = (D_{cl}/D_{Sun})^2 \times (I_{cl}/I_{Sun}) = (9.559 \times 10^8)^2 \times 8.318 \times 10^{-19} = 0.760$$

$$(M_{cl}/M_{Sun}) = (L_{cl}/L_{Sun})^{(1/3.8)} = 0.930$$

$$(t_{cl}/t_{Sun}) = (M_{cl}/M_{Sun})^{-2.8} = 1.224$$

$$t_{cl} = 1.224 \times t_{Sun} = 1.224 \times 8.2 \times 10^9 = \underline{\underline{10.0 \times 10^9 \text{ years}}}$$

By talking to experts at European Southern Observatory (Marina Rejkuba and Manuela Zoccali) it seems clear that the age determination we are suggesting is a rather simple method. They suggested an alternative method, which is to apply the following observed relation:

$$M_V(TO) = 2.70 \log(t) + 1.41,$$

where $M_V(TO)$ is the absolute magnitude of the turn-off point and t the age of the cluster in billions of years. By subtracting the distance modulus from $m_V(TO)$ we get the absolute magnitude of the turn-off point $M_V(TO)$:

$$m_V(TO) - (m_V - M_V) = 2.7 \log(t) + 1.41,$$

which reduces to:

$$t = 10^{[(m_V(TO) - (m_V - M_V)) - 1.41] / 2.7}$$

Resulting ages by calculating different sets of turn-off magnitude and distance by using the originally proposed method and by using the alternative method described above:

Measured Turn-off magnitude [m_V]	Calculated distance [pc]	Age, method 1 [billion years]	Age, method 2 [billion years]
18.7	4634	10.0	18.0
18.85	4634	11.1	20.5
18.5	4634	8.8	15.2
18.3	4900	7.0	11.6
18.3	4634	7.7	12.8
18.3	4500	8.0	13.5
18.7	6024 (no extinc.)	6.8	18.0

Bold figures are the best estimates from the literature.

Different methods of determining the age of globular clusters are described by Chaboyer et al. who find ages in the range between **11.5 and 15.9 x 10⁹ years** for M12.

Thanks to Anne Vørnholt Olesen, Ole Hjort Rasmussen, Helle and Henrik Stub, Denmark, and Johann Penzl, Germany, for pointing out these mistakes.