THE ESA/ESO ASTRONOMY EXERCISE SERIES

Student exercises in astronomy using
observations from the
NASA/ESA Hubble Space Telescope
and the ESO telescopes

Exercise 2

The Distance to M100 as Determined By Cepheid Variable Stars

Based on Observations with the NASA/ESA Hubble Space Telescope

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The Distance to M100
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Astronomy is an accessible and visual science, making it ideal for educational purposes. Over the last few years the NASA/ESA Hubble Space Telescope and the ESO telescopes at the La Silla and Paranal Observatories in Chile have presented ever deeper and more spectacular views of the Universe. However, Hubble and the ESO telescopes have not just provided stunning new images, they are also invaluable tools for astronomers. The telescopes have excellent spatial/angular resolution (image sharpness) and allow astronomers to peer further out into the Universe than ever before and answer longstanding unsolved questions.
The analysis of such observations, while often highly sophisticated in detail, is at times sufficiently simple in principle to give secondary-level students the opportunity to repeat it for themselves.

This series of exercises has been produced by the European partner in the Hubble project, ESA (the European Space Agency), which has access to 15% of the observing time with Hubble, together with ESO (the European Southern Observatory).

Figure 1 : The NASA / ESA Hubble Space Telescope
The NASA/ESA Hubble Space Telescope has presented spectacular views of the Universe from its orbit above the Earth.

Cosmology and distance measurements

How old is the Universe? How fast is it expanding? Will it one day start to contract? These are fundamental cosmological questions that have long awaited satisfactory answers.

The fate of the Universe is closely linked with the future behaviour/evolution of its expansion rate. If the expansion slows down sufficiently then the Universe may one day start to recontract. Observations currently suggest that it is more likely that the Universe will continue to expand forever.

The expansion makes all galaxies recede from a given observer (e.g. on Earth) and the further away they are, the faster they recede. The expression known as Hubble's law (formulated by Edwin Hubble in 1929) describes the relation between the distance of a given object and its recession velocity, V. Hubble's law is:

$$v = H_0 \cdot D$$

It states that the galaxies in our Universe are

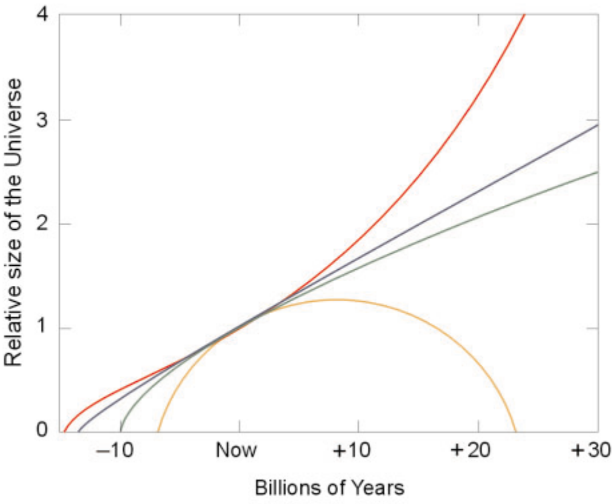

Figure 2: The Fate of the Universe
This graph relates the size of the Universe with time – in other words it shows how it expands and/or contracts with time. The different lines 'in the future' (to the right in the diagram) show different models for the fate of the Universe – an ever-expanding Universe or a contracting Universe.

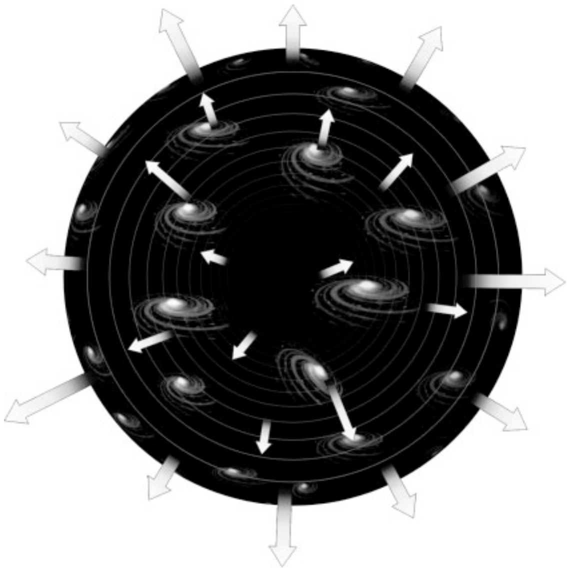

Figure 3: Receding Galaxies
This diagram illustrates how the galaxies recede from each other due to the expansion of the Universe.

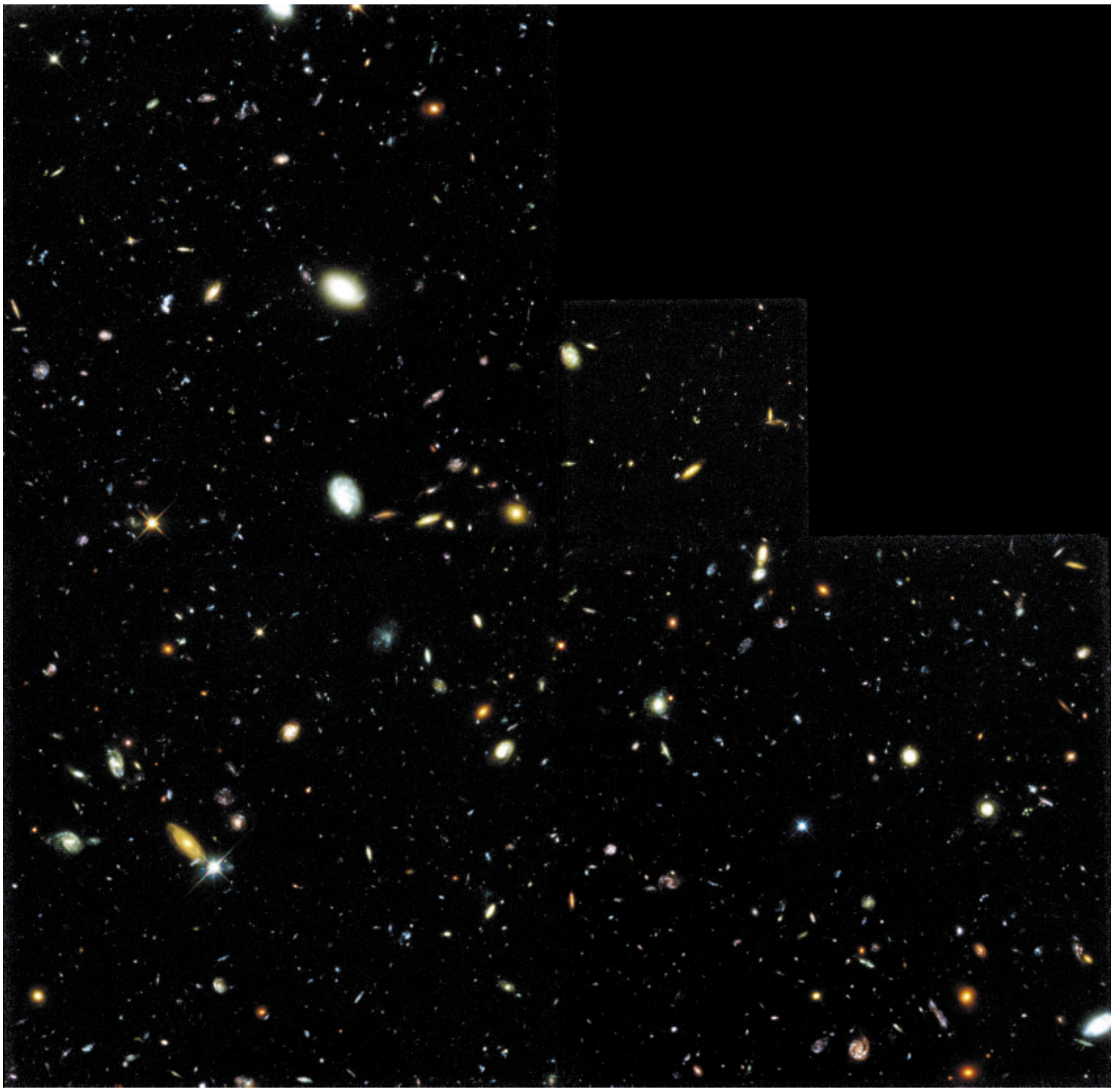

Figure 4: Remote Galaxies with High Redshifts
This image, taken by the Wide-Field and Planetary Camera (WFPC2) of the Hubble Space Telescope, shows many galaxies, billions of light years away. Most of the fuzzy patches are galaxies containing billions of stars. The galaxies in this image are receding from us at high velocities.

flying away from each other with a velocity, v, proportional to the distance, D, between them. H_0 is a fundamental property of the Universe – the Hubble constant – important in many cosmological questions and is a measure of how fast the Universe is expanding today.
The age of the Universe, t, can be approximated by the inverse (or reciprocal) of the Hubble constant H_0:

$t = 1/H_0$

The value of H_0 has enormous significance for estimates of the age of our Universe. But how do we measure it? To determine H_0, we 'simply' need to measure both the recession velocity, v, and the distance, D, for an object, usually a galaxy, or, even better, for many galaxies and find the average measurement.

The *recession velocity* is relatively easy to determine: we can measure the so-called redshift of the light from the galaxy. Redshift is a direct consequence of an object's motion away from us. It is a Doppler-shift of the light from the individual galaxies, resulting in a shift of the

Figure 5: Henrietta Leavitt
The understanding of the relative brightness and variability of stars was revolutionised by the work of Henrietta Swan Leavitt (1868-1921). Working at Harvard College Observatory, Leavitt calibrated the photographic magnitudes of 47 stars precisely to act as standard references or 'candles' for the magnitudes of all other stars. Leavitt discovered and catalogued over 1500 variable stars in the nearby Magellanic Clouds. From this catalogue, she discovered that brighter Cepheid variable stars take longer to vary, a fact used today to calibrate the distance scale of our Universe (Courtesy of AAVSO).

wavelength of the light from the galaxies towards the red end of the spectrum. As the wavelength of red light is longer than blue light, the wavelength of the light from the galaxies has increased during its journey to the Earth. The fractional change in wavelength due to the Doppler-shift is called the redshift and galaxies with a high redshift have high recession velocities.

Using Cepheids as distance estimators

Measuring the *distance* to an astronomical object is much more difficult and is one of the greatest challenges facing astronomers. Over the years a number of different distance estimators have been found. One of these is a class of stars known as Cepheid variables.

Cepheids are rare and very luminous stars that have a very regularly varying luminosity. They are named after the star δ-Cephei in the constellation of Cepheus, which was the first known example of this particular type of variable star and is an easy naked eye object.

In 1912 the astronomer Henrietta Leavitt (see Fig. 5) observed 20 Cepheid variable stars in the Small Magellanic Cloud (SMC). The small variations in distance to the individual Cepheid variable stars in the Cloud are negligible compared with the much larger distance to the SMC. The brighter stars in this group are indeed intrinsically brighter and not just apparently brighter, because they are closer. Henrietta Leavitt uncovered a relation between the intrinsic brightness and the pulsation period of Cepheid variable stars and showed that intrinsically brighter Cepheids have longer periods. By observing the period of any Cepheid, one can deduce its intrinsic brightness and so, by observing its apparent brightness calculate its distance. In this way Cepheid variable stars can be used as one of the 'standard candles' in the Universe that act either as distance indicators themselves or can be used to calibrate (or set the zero point for) other distance indicators. Cepheid variables can be distinguished from other variable stars by their characteristic light curves (see Fig. 6).

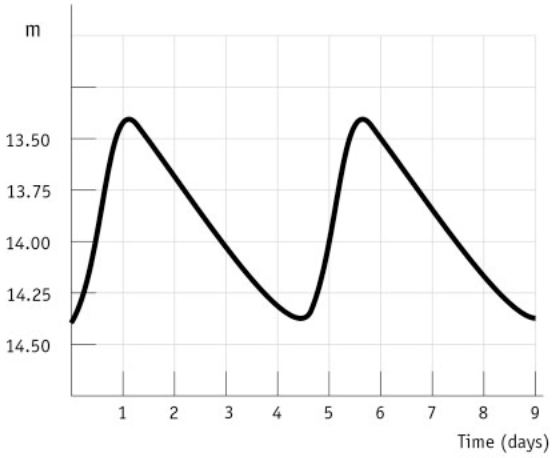

Figure 6: Typical Cepheid light curve
The light curve for a Cepheid variable star has a characteristic shape, with the brightness rising sharply, and then falling off much more gently. The amplitude of the variations is typically 1-2 magnitudes.

The most accurate measurements of both velocity and distance are naturally obtained for objects that are relatively close to the Milky Way. Before the NASA/ESA Hubble Space Telescope was available, ground-based observatories had detected Cepheid variables in galaxies with distances up to 3.5 Megaparsecs (see the definition of Megaparsecs in the Mathematical Toolkit) from our own Sun. However, at this sort of distance, another velocity effect also comes into play. Galaxies attract each other gravitationally and this introduces a non-uniform component to the motion that affects our measurements of the uniform part of the velocity arising

from the expansion of the Universe. This non-uniform part of the velocity is known as the peculiar velocity and its effect is comparable with the expansion velocity in our local part of the Universe. In order to study the overall expansion of the Universe, it is necessary to make reliable distance measurements of more distant galaxies where the expansion velocity is significantly higher than the peculiar velocity. Hubble has measured Cepheid variables in galaxies with distances of up to ~20 Megaparsecs.

Before Hubble made these measurements astronomers argued whether the Universe was 10 or 20 billion years old. Now the agreement is generally much better — the age of the Universe is believed to be somewhere between 12 and 14 billion years.

One of the Hubble's Key Projects had as a long-term goal a more accurate value for the Hubble constant and the age of the Universe. Eighteen galaxies located at different distances have been monitored to reveal any Cepheid variables. One of these galaxies is M100.

Figure 7: The Spiral Galaxy M100

If we could observe our galaxy, the Milky Way, face on from an extragalactic space vessel, its shape would be similar to the Spiral Galaxy M100.

Spiral galaxies are rich in dust and gas. The dust shows up in this image as dark lanes running between the majestic spiral arms.

M100 is a popular target for amateur astronomers and is located in the spring sky in the direction of the constellation of Coma Berenices. The image was taken with Hubble's Wide Field and Planetary Camera 2. Blue colours correspond to regions with young hot stars.

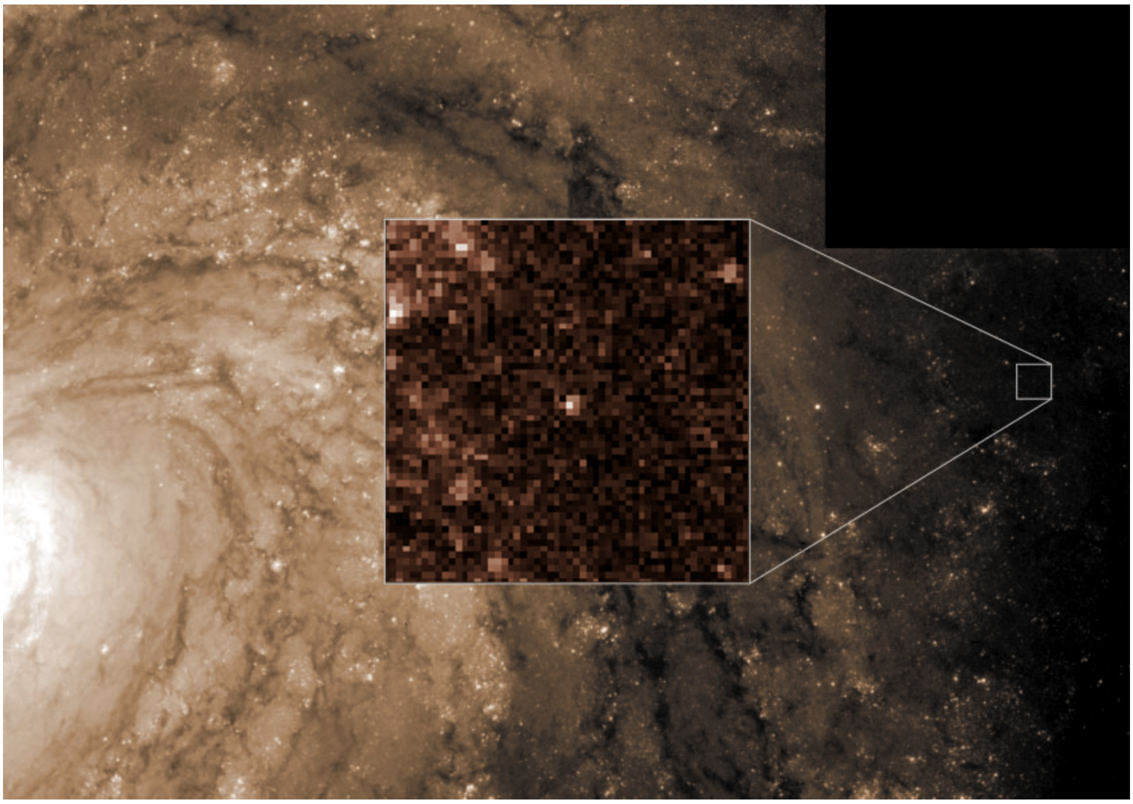

Figure 8: Hubble tracks down Cepheid variable stars in M100
Hubble's high resolution camera detected and picked out one of the Cepheid variable stars used in this exercise. The star is located in a star-forming region in one of the galaxy's spiral arms (the star is at the centre of the box).

M100 a Grand Spiral

The galaxy M100 is a magnificent spiral galaxy in the large Virgo cluster of galaxies. The Virgo cluster contains 2,500 galaxies. M100 is a rotating system of gas, dust and stars similar to the Milky Way and is viewed face on. The name M100 stems from the fact that it is number 100 in the Messier catalogue of non-stellar objects.

M100 is one of the more distant galaxies where accurate measurements of Cepheid variables have been made. This exercise is based on Hubble's images and data from this galaxy.

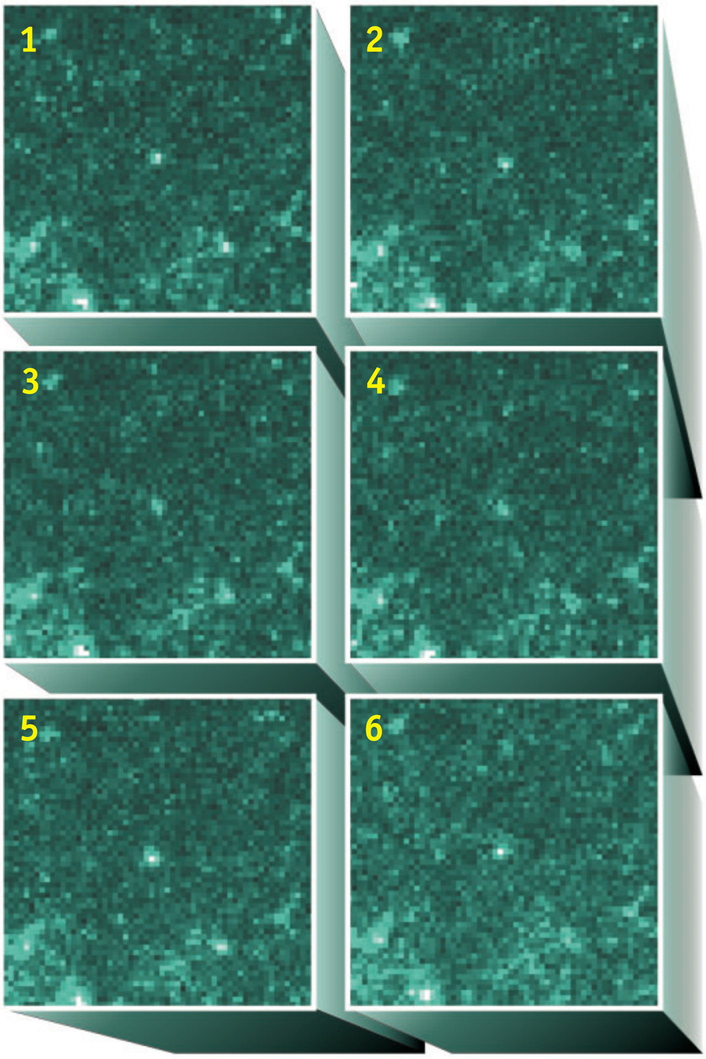

Figure 9: A Cepheid variable in M100
Six images taken at different times depicting one of the Cepheid variable stars in the galaxy M100 are shown. The Cepheid is in the centre of each frame. It is clear that the Cepheid varies in brightness over time.

Measurements and calculations

The Period-Luminosity relation for Cepheid variables has been revised many times since Henrietta Leavitt's first measurements. Today the best estimate of the relation is:

$$M = -2.78 \log(P) - 1.35$$

where M is the absolute magnitude of the star and P is the period measured in days.

Light curves for the 12 Cepheids in M100 that have been measured with Hubble are shown on pages 9 and 10.

Task 1

? Using the information in these curves, calculate the absolute magnitude M for the 12 stars.

Our goal is to calculate the distance to M100. If you remember the distance equation, you will know that the absolute magnitude alone is not enough to calculate the distance — you also need the apparent magnitude.

Apart from problems in measuring the amount of light received accurately and calibrating the magnitudes that were measured, for a hundred years astronomers have discussed which apparent magnitude, m, to use in the distance equation for a Cepheid that is actually varying in magnitude.

Task 2

? Think about a method to estimate the apparent magnitude, m, using the curves.

At the beginning of the 20th century astronomers measured the minimum apparent magnitude (m_{min}) and the maximum apparent magnitude (m_{max}) and then took the average (<m>) of the two.

If you do that – or use your own method – you now have all the information you need to calculate the distance to M100.

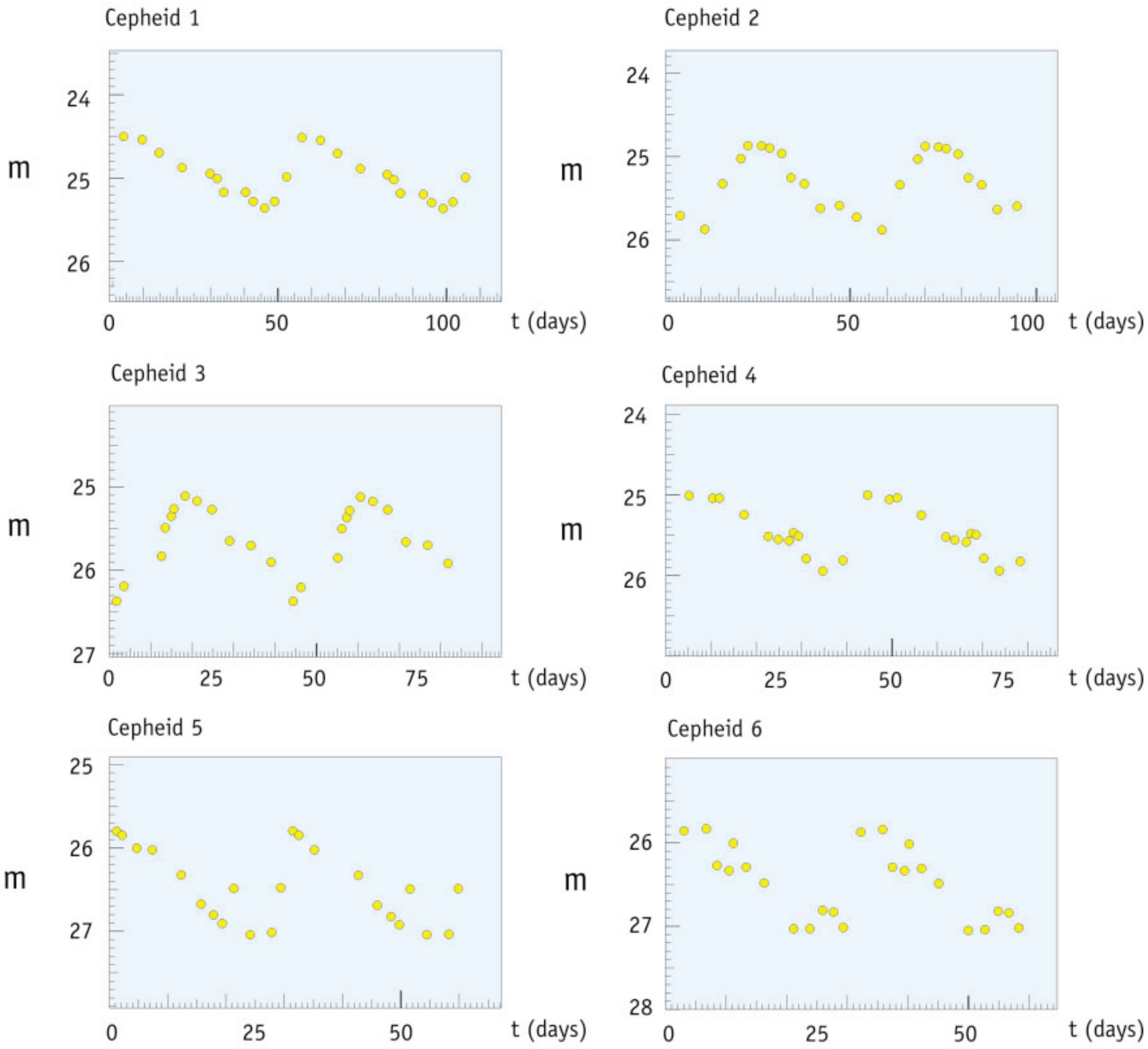

Figure 10: Cepheid light curves
Light curves for the twelve Cepheid variables in M100 that have been observed with Hubble. The absolute magnitude, M, is determined from the period of the Cepheids. Adapted from Freedman et al. (1994).

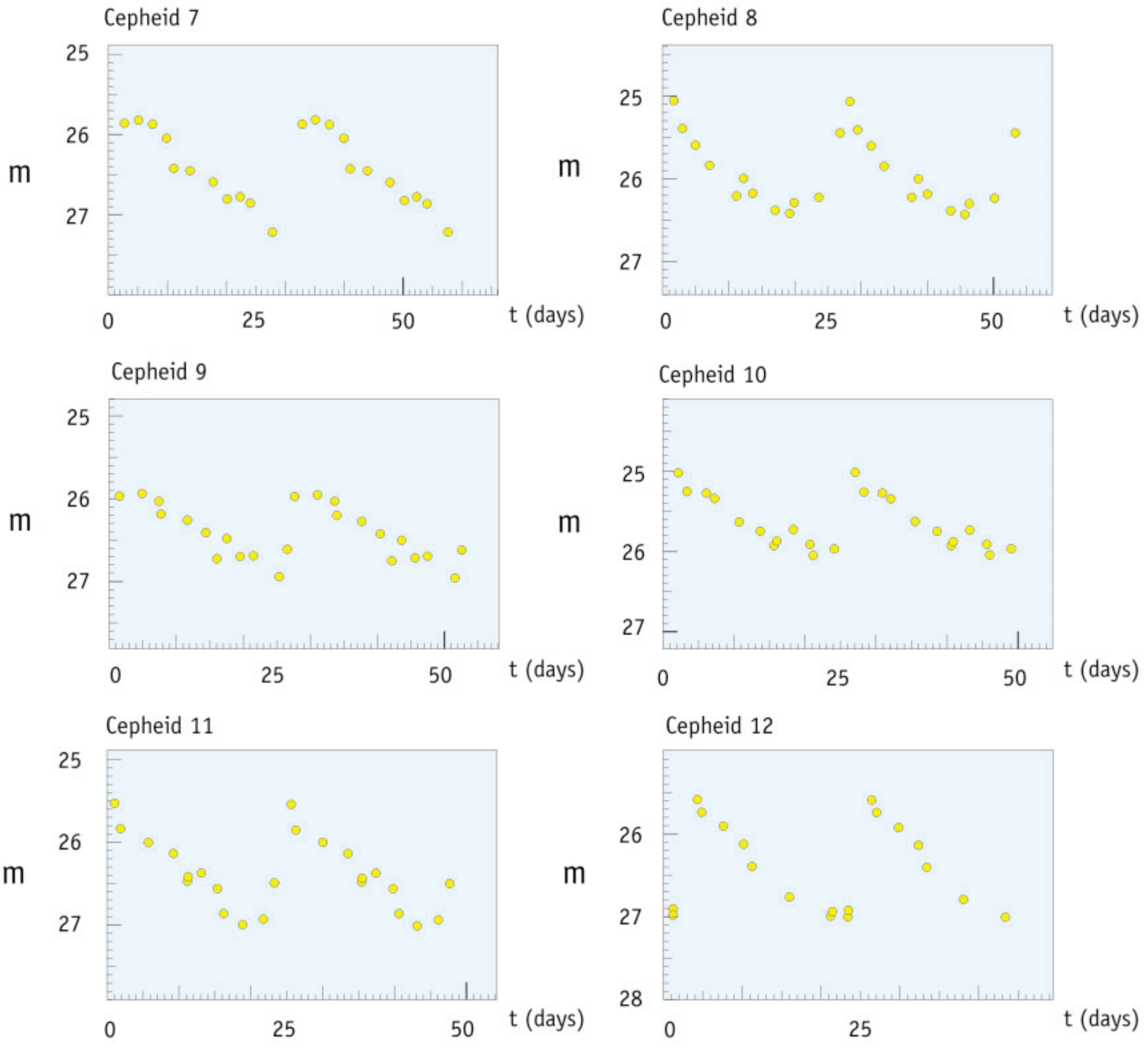

Figure 10 (continued): Cepheid light curves

Task 3

? Calculate <m> and D (in Mpc) for each Cepheid.

You could, of course, just do the same calculation twelve times, but you can reduce the amount of work by, for example, writing a small program for a calculator or using a spreadsheet.

Task 4

? Consider the probable reasons why you do not find exactly the same distances for the different Cepheid variables.

Task 5

? Now you have calculated the distance to the twelve different Cepheid variable stars in M100. Does that give you the distance to M100?

Tasks

? Could the fact that the twelve stars have different positions in M100 be the reason for the variation in the distances of the twelve stars?

? Find out how large the Milky Way is (you can, for instance, look in an astronomy book or on the Internet). Assume that the size of M100 is of the same order. Now think about the previous question once again.

Task 6

? Calculate the mean value of the distances to the twelve Cepheid stars and consider this as the distance to M100.

? In the original scientific paper using the Hubble measurements, the distance to M100 was calculated as 17.1 ± 1.8 Megaparsecs. The presence of interstellar dust was taken into account in determining this value. Compare your results with that distance.

Task 7

As you may remember from the introduction (p. 5), the recession velocity, v, of a galaxy like M100, together with information about its distance can give you a value for the general expansion velocity of the Universe as described by Hubble’s constant, H_0. H_0 is expressed in units of km/s/Mpc. The recession velocity of the Virgo Cluster, of which M100 is a member, has been measured earlier to be 1400 km/s (Freedman et al., 1994).

? Calculate Hubble’s constant using this v, and the average of your distance measurements as D.

Task 8

? Assuming that the age of the Universe, t, is given by $t = 1/H_0$, calculate a value for the age of the Universe. Remember to convert to the correct units. How much older is this than the age of the Earth?

Further Reading

Scientific Papers

- Freedman, W.L., Madore, B.F., Mould, J.R., Ferrarese, L.; Hill, R., Kennicutt, R.C., Jr., Saha, A., Stetson, P.B., Graham, J.A., Ford, H., Hoessel, J.G., Huchra, J., Hughes, S.M., and Illingworth, G.D., 1994, Nature, 371, 757-762.: *Distance to the Virgo cluster galaxy M100 from Hubble Space Telescope observations of Cepheids.*

See also the Links on:
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Teacher's Guide

Quick Summary

In this exercise we measure the period and apparent magnitudes of Cepheid variables in the galaxy M100. The absolute magnitude is derived using the Period-Luminosity relation and the distance to M100 can then be determined using the distance relation. Finally we calculate a value for the Hubble constant (using a value for the recession velocity of M100 observed by other scientists) and estimate the age of the Universe.

This teacher's guide contains solutions to the problems together with a discussion of the approximations and simplifications made in the exercise.

The hypothesis that the Universe has expanded since the Big Bang at a constant rate is, strictly speaking, only correct in certain cosmological models. Such an expansion is really only possible if the Universe contains very little matter, as all matter, whether visible or dark, interacts gravitationally to slow down the expansion rate. Recent results have led to no firm conclusions concerning the expansion rate of the Universe, and so we can consider the expression used in these tasks as a simple, but reasonable approximation.
Note that, according to recent cosmological models, the Universe underwent a phase of decreasing expansion (due to the gravitational effects of dark and normal matter) that lasted about 5 billion years after the Big Bang. Since then the Universe appears to have entered a period with an accelerating expansion rate where a mysterious 'repulsive gravitation' has taken over. This force is also known by the names 'Dark Energy' or 'quintessence' (the fifth element).

Tasks 1, 2 and 3

Using the method suggested in task 2 and simple measurements with a ruler on the paper copy we obtain the following results:

Cepheid number	t2	t1	period = t2-t1	M	m max	m min	m average	D Mpc	D average Mpc
1	100.0	46.5	53.5	-6.15	24.50	25.30	24.90	16.25	19.85
2	58.5	11.0	47.5	-6.01	24.90	25.90	25.40	19.15	
3	61.0	18.5	42.5	-5.88	25.10	26.40	25.75	21.15	
4	74.0	35.0	39.0	-5.77	25.00	25.95	25.48	17.77	
5	50.0	19.0	31.0	-5.50	25.80	27.05	26.43	24.22	
6	50.0	21.0	29.0	-5.42	25.80	27.10	26.45	23.61	
7	35.0	4.5	30.5	-5.48	25.80	27.20	26.50	24.85	
8	46.0	19.0	27.0	-5.33	25.05	26.40	25.73	16.25	
9	31.0	5.0	26.0	-5.28	25.90	27.00	26.45	22.22	
10	27.0	2.5	24.5	-5.21	25.00	26.10	25.55	14.20	
11	43.0	19.0	24.0	-5.19	25.55	27.00	26.28	19.61	
12	38.0	16.0	22.0	-5.08	25.60	27.00	26.30	18.90	

As M100 is very distant, other methods (such as, for instance, plotting m(P)) do not work very well. We have chosen to supply the Period-Luminosity relation instead of allowing students to derive the two coefficients in that equation for themselves. As a result the exercise is accessible to a larger group of students — something we consider a positive advantage (within reason of course :-)).

Tasks 4

The reason that first springs to mind for any deviation in the results is simply the normal uncertainty in the measurements. Measurements of this kind, made by hand, are not very precise. The accuracy could be improved by using more refined methods of measurement.
Alternatively there may be two different classes of Cepheids that have slightly different characteristics.

Task 5

Yes, based on the (relatively) large sample of Cepheids, we now have a reasonable estimate for the distance to M100.
No, the size of a galaxy is small compared with the distance to M100.
The Milky Way is approximately 25 kpc in diameter. The answer to the previous question is definitely still no.

Task 6

With the crude methods used here, a value of 19.8 Mpc is quite reasonable.
The question is posed to make students realise that uncertainties are part and parcel of many natural sciences and certainly so in astronomy.

Task 7

$H_0 = v/D = 1400/19.85 = \mathbf{70.53\ km/s/Mpc}$

This value is within the accepted range. Generally H_0 is considered to lie between 60 and 80 km/s/Mpc.

Task 8

Using the conversion factor for Mpc to km, we get for $H_0 = 2.286 \times 10^{-18}\ \text{s}^{-1}$.

$t = 1/H_0 = 4.375 \times 10^{17}\ \text{s} = \mathbf{13.87 \times 10^{9}\ years}$

This is about three times the age of the Earth (~4.6 billion years). This question was posed to try to make students relate the age of the Universe to something they might know beforehand.

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