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THE RING AROUND SUPERNOVA 1987A REVISITED. I. ELLIPTICITY OF THE RING

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ABSTRACT

Panagia et al. (1991) have measured the distance to the Large Magellanic Cloud, $D_{\rm LMC} = 50.1 \pm 3.1$ kpc from the fluorescence of the ring around SN 1987A, which was assumed to be circular. I recalculate $D_{\rm LMC}$ using the supernova-ring method and the data of Panagia et al. both with and without the assumption that the ring is circular. For a circular ring, I find $D_{\rm LMC} = 53.2 \pm 2.6$ kpc, 3 kpc larger than the result of Panagia et al. For a ring of intrinsic eccentricity e the distance is smaller than for a circular ring by a fraction $\sim 0.4e^4$, i.e., only 1% for $e = 0.4$.

Subject headings: cosmology: theory — Magellanic Clouds — supernovae: individual (SN 1987A) — supernova remnants

1. INTRODUCTION

Panagia et al. (1991) have derived a distance to SN 1987A in the Large Magellanic Cloud (LMC) from the light curve of the ring illuminated by the supernova. They find a distance of 51.2 ± 3.1 kpc. Correcting for the position of the supernova relative to the LMC center of mass, they find a distance to the latter of

$$D_{\rm LMC} = 50.1 \pm 3.1 \text{ kpc} \quad \text{(Panagia et al.)}. \tag{1.1}$$

The determination by Panagia et al. (1991) of the distance to the supernova ring rests on three assumptions.

1. The observed ring of illuminated gas is indeed a thin planar structure, rather than a density caustic in a three-dimensional (e.g., ellipsoidal) structure.
2. The caustics in the ionized emission curves seen at 83 ± 6 and 413 ± 24 days identify the extreme light travel times for the paths going from supernova to ring to observer.
3. The ring is actually circular and appears elliptical because it is seen in projection.

Once these three assumptions are accepted, only geometry enters the calculation. The systematic errors are therefore small compared to the statistical errors and the measurement should be taken at face value.

In § 2, I review the Panagia et al. (1991) calculation. I show that the first assumption is well-founded in the data. The second assumption is qualitatively well-founded. That is, the caustics do indeed represent the extreme light travel times; however, the specific estimates and error bars of these caustics may require some revision. The third assumption is not well-founded. Thus, the major uncertainty concerning the supernova-ring measurement of $D_{\rm LMC}$ is systematic. The question is, how much does the distance estimate change if one relaxes the assumption of circularity? I address this question in the present paper. For this purpose, I adopt the light travel times of Panagia et al. (1991). In Paper II of this series I will reanalyze the light-curve data and derive new estimates for the light travel times and their errors. Using these new estimates and the general formulae derived in the present paper, I will then present a revised estimate of $D_{\rm LMC}$. In Paper III of this series, I will discuss the implications of this measurement for cosmology.

In § 3, I recalculate $D_{\rm LMC}$ under the assumption of a circular ring and find

$$D_{\rm LMC} = 53.2 \pm 2.6 \text{ kpc} \quad \text{(circular ring)}. \tag{1.2}$$

The difference between this value and that given by Panagia et al. (eq. [1.1]) is due to two approximately equal effects. First, I have made a more careful analysis of how the observations of the ring should be combined to obtain a best estimate of the distance to SN 1987A. Second, I have adopted the value used by Jacoby et al. (1992) for the relative distances of SN 1987A and the center of mass of the LMC. In § 4, I relax the assumption that the ring is circular. I find that the distance estimate is thereby reduced by a fraction $\sim 0.4e^4$, where e is the intrinsic eccentricity of the ring. For small to moderate eccentricities ($e < 0.4$), this correction is negligibly small, $< 1\%$.

2. THE PANAGIA ET AL. MEASUREMENT

The measurement of the distance to SN 1987A by Panagia et al. (1991) rests on the three assumptions enumerated above in § 1. Briefly, the argument given by Panagia et al. is as follows. For a circular ring, the light travel times to the far and near sides of the apparent minor axis (less the light travel time directly from the supernova) are

$$t_+ = \frac{d}{2c}(1 + \sin i), \tag{2.1}$$

$$t_- = \frac{d}{2c}(1 - \sin i), \tag{2.2}$$

where d is the physical diameter of the ring and i is the angle of inclination of the plane of the ring to the line of sight. The first term in these equation is the travel time from the center to the circumference of the ring. The second term is the travel time from the plane of the ring to the plane of the sky at the ring circumference. From equations (2.1) and (2.2), one finds

$$\sin i = \frac{t_+ - t_-}{t_+ + t_-}. \tag{2.3}$$

Alternatively, one may estimate the angle of inclination from the apparent axis ratio of the ring

$$\cos i = \frac{\theta_-}{\theta_+}, \tag{2.4}$$

where $\theta_{\pm}$ are the major and minor angular diameters of the apparent ellipse.

Panagia et al. model the light curves to determine $t_{\pm}$ and find

$$t_{+} = 413 \pm 24 \text{ days}, \quad t_{-} = 83 \pm 6 \text{ days}. \qquad (2.5)$$

From these and equation (2.3), they make one measurement of the inclination, $i = 42° \pm 5°$. They then use the ellipse diameters measured by Jakobsen et al. (1991),

$$\theta_{+} = 1''.66 \pm 0''.03, \quad \theta_{-} = 1''.21 \pm 0''.03, \qquad (2.6)$$

and equation (2.4) to make another measurement of the inclination, $i = 43° \pm 3°$. (I estimate smaller error bars on both inclination measurements; see § 3). They combine the two measurements to form an average value $\langle i \rangle = 42°.8 \pm 2°.6$, substitute into equation (2.1) to find d, and compute the distance to SN 1987A, $d/\theta_{+} = 51.2 \pm 3.1$ kpc.

Before proceeding to an examination of the assumptions that underlie this calculation, I note that equations (2.1) and (2.2) play symmetric roles in the distance derivation and either might have been used in the penultimate step to derive the physical ring diameter, d. If Panagia et al. had used equation (2.2), they would have found $d/\theta_{+} = 54$ kpc, almost 1 σ higher than the value derived using equation (2.1). I return to this point in the next section.

I now turn to the assumptions. The ring appears planar, but as Dwek & Felten (1992) have emphasized, one should be cautious. Planetary nebulae are ellipsoidal shells and often appear as ellipsoidal rings in projection. Crotts & Heathcote (1991) have measured the redshift of the ring emission and find expansion (or contraction!—see McCray & Lin 1993) along the minor axis but essentially no expansion along the major axis. This is consistent with a ring seen with an inclination vector that is approximately aligned with the apparent minor axis but not with an ellipsoid. Dwek and Felten adduce a second argument that the ring structure is indeed planar, namely, that there is a delay of ~80 days between the supernova and the beginning of the fluorescent emission. This is characteristic of an open topology, such as a ring tilted to the line of sight, but not a closed topology such as an ellipsoid.

Another nonplanar geometry should also be considered. The ring may well be a "belt" around the center of the hourglass in the Napoleon's Hat nebula (Podsiadlowski, Fabian, & Stevens 1991). In this case, there might be a near-cylinder of gas extending out of the ring plane. However, since the ring is inclined at ~45°, such a cylinder would appear almost exactly like a ring with finite thickness in the plane, and the effect on the timing arguments would likewise be almost exactly the same. Thus, to the extent that this goemetry is allowed, it has no special consequences for the problem.

Once the geometry is established as planar, the meaning of the caustics seen in the light curve (see Fig. 2 of Panagia et al.) is clear: a burst of light incident on an arbitrary, smooth, convex, *reflection* nebula will always produce caustics in the light curve at the extreme times of reflection. The supernova ring does not reflect, but rather fluoresces, and therefore the (theoretical) reflection light curve must be convolved with a transfer function which characterizes the fluorescence. However, only a pathological transfer function could create, destroy, or move the caustics in the underlying reflection light curve. The one possibility that really must be considered is that the fluorescence has an extremely slow start-up and peaks very quickly well after the burst of illumination. Such a possibility must be considered for the N v line, which is permitted and hence may be optically thick. However, the remaining three lines measured by Panagia et al. (N III], N IV], and C III]) are semiforbidden and thus optically thin. Since the recombination time is about four orders of magnitude longer than the span of the observations, there would appear to be no physical mechanism that could produce a delayed start up for the semiforbidden lines. Thus, the assumption that the caustics represent extreme light travel times is qualitatively well-founded.

Dwek & Felten (1992) agree that caustics represent extreme travel times but have raised an important question regarding the accuracy of the specific estimates (2.5). They show that for a ring the reflection function (which is convolved with the transfer function to obtain the light curve) is given by

$$R[x(t)] = \frac{A}{\pi} \frac{\Theta(1-x)\Theta(1+x)}{\sqrt{1-x^2}}, \quad x(t) \equiv \frac{2t - t_{+} - t_{-}}{t_{+} - t_{-}}, \qquad (2.7)$$

where Θ is the Heaviside step function and A is the normalization. By contrast, Panagia et al. use the form

$$R(x) = \frac{A}{2}\Theta(1-x)\Theta(1+x) \quad \text{(approximation)}. \qquad (2.8)$$

Panagia et al. argue that their approximation should not affect the location of the caustics and Dwek & Felten offer no specific argument to contradict them. However, in Paper II, I will show that equation (2.7) yields a much sharper peak at the second caustic than does equation (2.8). This peak is clearly present in the best data set (N III]) in a cluster of six points at ~390 days. A peak is also visible at ~390 days in the N IV] data. The Panagia et al. fit at 413 days appears reasonable only because they have been boxcar-smoothed over eight points and only because the left-hand-side of the peak of their curve is much broader than the left-hand-side of the true peak. Hence, Dwek & Felten may be correct in their contention that using equation (2.7) can have a significant impact on the result. Finally, I note that in their Table 1, Panagia et al. report four independent measurements and errors for each of the quantities, t_{+} and t_{-}. Taken together, these results have a χ^2 per degree of freedom of 0.05 for 6 degrees of freedom. If the errors have been correctly estimated, then the probability for such a low χ^2 is 0.0005.

In brief, the data should be reanalyzed taking account of equation (2.7) and the possibility of a finite rise time for the N v line. Nevertheless, since the basic assumption that the caustics represent extreme travel times is correct, one can use this assumption to derive formulae for the distance to the supernova. If the estimates for $t_{\pm}$ and their errors are subsequently revised, the new values can be substituted into these formulae.

The third assumption, that the ring is intrinsically circular, is less secure than the other two. Panagia et al. give two arguments for circularity. First, they say that "it is *physically* very hard to produce a high-eccentricity structure *centered* on its source (p. L23)." Second, they point to the agreement in the inclinations as calculated from equation (2.3) and (2.4) as being consistent with the hypothesis of a circle. Neither of these arguments is compelling.

The fact that the inclinations as calculated from equations (2.3) and (2.4) are consistent with a circular ring does not make the ring circular. I construct explicit counterexamples below. The fact that we cannot think of a mechanism to produce an ellipse does not mean that an ellipse is excluded; nature is more clever than we are. For example, the gas in the ring is clumpy, which may result from inhomogeneities in the medium into which it is expanding. Such inhomogeneities might deform an initially circular ring to be elliptical.

3. CALCULATION FOR A CIRCULAR RING

Here I recalculate the distance to SN 1987A under the assumption that the ring is circular. In doing so, I introduce most aspects of the formalism that will be required for the noncircular case. The value that I derive for the distance is 1.5 kpc larger than that derived by Panagia et al.

First, I define four new multiplicative combinations of the measured quantities:

$$t_\times \equiv \sqrt{t_+ t_-} = 185 \pm 9 \text{ days}\,, \quad (3.1)$$

$$\theta_\times \equiv \sqrt{\theta_+ \theta_-} = 1''.42 \pm 0''.02\,, \quad (3.2)$$

$$\eta_t \equiv \frac{t_-}{t_+} = 0.201 \pm 0.019\,, \quad (3.3)$$

and

$$\eta_\theta \equiv \frac{\theta_-}{\theta_+} = 0.729 \pm 0.022\,. \quad (3.4)$$

The correlation coefficients of $t_\times$ with η_t and $\theta_\times$ with η_θ are 0.15 and 0.3, respectively. The four other pairs of quantities are independent.

From equations (2.1) and (2.2), one finds that $t_\times = (d/2c) \cos i$. Equation (2.4) implies that $\theta_\times = (d/D_{SN}) \sqrt{\cos i}$, where D_{SN} is the distance to the supernova. Hence,

$$D_{SN} = D_\times G(i)\,, \quad (3.5)$$

where

$$D_\times \equiv c\,\frac{t_\times}{\theta_\times} = 22.6 \pm 1.1 \text{ kpc}\,, \quad (3.6)$$

and

$$G(i) = 2\sqrt{\sec i}\,. \quad (3.7)$$

Under the assumption that the ring is circular, the inclination can be measured by two independent methods. First, from η_θ, using equation (2.4),

$$i = \cos^{-1} \eta_\theta = 43^\circ.2 \pm 1^\circ.8\,, \quad (3.8)$$

and second, from η_t, using a transformation of equation (2.3),

$$i = \frac{\pi}{2} - 2 \tan^{-1}\sqrt{\eta_t} = 41^\circ.7 \pm 2^\circ.0\,. \quad (3.9)$$

In both cases, I have determined the error bars by using the chain rule. Since the methods are independent, they can be combined to yield $i = 42^\circ.5 \pm 1^\circ.4$, or

$$G(i) = 2.329 \pm 0.025\,. \quad (3.10)$$

The quantities $D_\times$ and $G(i)$ are virtually independent (correlation coefficient $\sim$0.01), so the errors in equation (3.5) can be combined in the standard way to yield

$$D_{SN} = 52.7 \pm 2.6 \text{ kpc}\,. \quad (3.11)$$

Note that this result is 1.5 kpc larger than that found by Panagia et al. based on the same data. The reason for the difference is that by using equation (2.1) alone rather than averaging over equations (2.1) and (2.2), Panagia et al. in effect gave higher statistical weight to some data than others. By transforming to the new variables, $t_\times$, $\theta_\times$, η_t, and η_θ, I have been able to carry out the calculation with all measured quantities weighted according to their quoted errors.

To find the distance to the LMC, Panagia et al. assumed that SN 1987A lies 1.1 kpc farther from us than does the center of mass of the LMC. However, Jacoby et al. (1992) point out that the eastern side of the LMC disk is known to be closer. Both the massive star that was progenitor to SN 1987A and the entire 30 Dor region in which it lies are Population I objects and therefore very likely lie in the plane of the LMC, implying that SN 1987A is 0.5 kpc *closer* than the LMC center of mass.[1] I adopt this correction and find

$$D_{LMC} = 53.2 \pm 2.6 \text{ kpc}\,. \quad (3.12)$$

4. CALCULATION FOR AN ELLIPTICAL RING

The formalism developed in the previous section for a circular ring can be generalized to the elliptical case. The resulting equations can be solved analytically in limit of small eccentricity, e, that is, for $e^2 \ll 1$. To first order in e^2, the distance measurement is unchanged from the circular case. To second order, a finite eccentricity moves the LMC closer by a fractional amount $\sim 0.4(e^2)^2$. This systematic effect becomes significant (relative to the statistical errors) when $e \gtrsim 0.55$, that is, for axis ratios $b/a \lesssim 0.85$. A numerical solution confirms these analytic results.

Suppose that the ring has major and minor semiaxes, a and b, and that the unit vector normal to the plane in which it lies is inclined to the line of sight at an angle i. Let ϕ be the position angle of the minor axis relative to the line in the plane which is maximally inclined to the line of sight. The geometry of the ellipse is then characterized by its distance D_{SN}, its physical scale, a, and three dimensionless parameters, i, ϕ, and e, where

$$e^2 \equiv 1 - \frac{b^2}{a^2}\,. \quad (4.1)$$

Projected on the sky, the ellipse will appear as a smaller ellipse. I label the projected major and minor semiaxes, a' and b'. The product of these axes is proportional to the area of the projected ellipse, that is,

$$a'b' = ab \cos i\,. \quad (4.2)$$

After some algebra (see Appendix), one finds that the ratio of the axes is

$$\frac{b'}{a'} = f(i, \phi, e) - \sqrt{f^2(i, \phi, e) - 1}\,, \quad (4.3)$$

where

$$f(i, \phi, e) \equiv \frac{1}{4}\left(\frac{a}{b} + \frac{b}{a}\right)(\sec i + \cos i) + \frac{1}{4}\left(\frac{a}{b} - \frac{b}{a}\right)(\sec i - \cos i)\cos(2\phi)\,. \quad (4.4)$$

[1] A large amount of material lies $\sim$300 pc in front of the supernova, and therefore one might plausibly argue that this material represents the plane of the LMC, placing SN 1987A slightly farther back. However, this discrepancy is small compared to the other errors in the problem.

Let γ be the position angle of an arbitrary point on the ellipse. The distance from the supernova to that point is then given by

$$r^2 = a^2 \sin^2 (\gamma - \phi) + b^2 \cos^2 (\gamma - \phi) . \quad (4.5)$$

The light travel time from the supernova to an arbitrary point to the observer (less the time of travel directly from the supernova) is

$$t = \frac{r}{c} (1 + \sin i \cos \gamma) . \quad (4.6)$$

This equation may be rewritten as

$$t = \frac{\sqrt{ab}}{c} g(i, \phi, e, \gamma) , \quad (4.7)$$

where

$$g(i, \phi, e, \gamma) \equiv \sqrt{1 + \xi}(1 + \sin i \cos \gamma) \times \sqrt{1 - \frac{e^2}{2 - e^2} \cos (2\gamma - 2\phi)} , \quad (4.8)$$

and

$$\xi \equiv \frac{(a - b)^2}{2ab} \sim \frac{e^4}{8} . \quad (4.9)$$

The caustics in the light curve occur at extreme times, which are found by differentiating equation (4.7) and setting $dt/d\gamma = 0$. Thus, the caustics lie at the γ which solve the equation,

$$\sin \gamma = \frac{e^2}{2 - e^2} [\csc i \sin (2\gamma - 2\phi) + \sin (3\gamma - 2\phi)] . \quad (4.10)$$

This equation has at least two solutions. The cases that have more than two solutions correspond to light curves with more than two caustics. Since the actual light curve has only two caustics, I will ignore the more complicated cases.[2] I label the coordinates of the two solutions $\gamma_\pm$ and define $g_\pm$ by

$$g_\pm(i, \phi, e) \equiv g(i, \phi, e, \gamma_\pm) . \quad (4.11)$$

The measurable quantities defined in the previous section may now by written in terms of the parameters of the ellipse:

$$t_\times = \frac{\sqrt{ab}}{c} \sqrt{g_+ g_-} , \quad (4.12)$$

$$\theta_\times = 2 \frac{\sqrt{ab}}{D_{SN}} \sqrt{\cos i} , \quad (4.13)$$

$$\eta_t = \frac{g_-}{g_+} , \quad (4.14)$$

and

$$\eta_\theta = f - \sqrt{f^2 - 1} , \quad (4.15)$$

[2] Numerically, I find that the regions of parameter space with more than two caustics are adjacent to the regions that are permitted with low probability. They have lower inclinations than the permitted regions. An easy-to-visualize case of extra caustics was pointed out to me by the referee: consider a high-eccentricity ellipse whose minor axis is parallel to the line to sight. The first caustic comes from the near side and the second from the far side. A final caustic comes from points on the major axis.

where f and $g_\pm$ are given by equation (4.4) and (4.11). Hence, one may generalize equation (3.5) and write

$$D_{SN} = D_\times G(i, \phi, e) , \quad (4.16)$$

where

$$G(i, \phi, e) \equiv 2 \sqrt{\frac{\cos i}{g_+ g_-}} . \quad (4.17)$$

The problem can now be solved by using the observed values of η_t and η_θ together with equations (4.14) and (4.15) to constrain the ellipse parameters i, ϕ, and e. For the allowed parameters, one may evaluate G and thus the distance D_{SN} using equations (4.16) and (4.17).

Of course, it is impossible to evaluate the three ellipse parameters with only two equations. Even with perfect data there would be one degree of degeneracy in the allowed range of these parameters. However, the primary interest is not in these parameters per se but only in the distance that they imply. In order to explore the nature of this degeneracy and its implications for D_{SN}, I begin with a perturbative solution to the equations, expanding in the parameter $\epsilon = e^2$. For simplicity, I assume that the data are perfect and that the measured values of η_t and η_θ imply the same inclination i_0 when a circular ring is assumed (see eqs. [3.8] and [3.9]).

4.1. *Zeroth Order*

From equation (4.10), $\gamma_{+,0} = 0$ and $\gamma_{-,0} = \pi$. The position angle ϕ is indeterminate, that is, all values are equally acceptable. For purposes of continuity with the first order equation, however, I choose $\phi_0 = \pi/4$, $\cos (2\phi_0) = 0$. From equation (4.8), $g_{+,0} g_{-,0} = \cos^2 i_0$.

4.2. *First Order*

From equation (4.10),

$$\gamma_{+,1} = -\frac{\epsilon}{2} (\csc i_0 + 1), \quad \gamma_{-,1} = \pi + \frac{\epsilon}{2} (\csc i_0 - 1) . \quad (4.18)$$

Note that for an elliptical ring, the caustics do not come from opposite sides of the ring, that is, $\gamma_{-,1} - \gamma_{+,1} \neq \pi$. From equation (4.8), I find

$$\frac{g_{-,1}}{g_{+,1}} = \frac{1 - \sin i_1}{1 + \sin i_1} , \quad (4.19)$$

which, together with equation (4.14), implies that to first order there is no change in the inclination,

$$i_1 = i_0 . \quad (4.20)$$

Equations (4.4) and (4.15) require that

$$\cos (2\phi_1) = -\frac{\epsilon}{4} (2 \csc^2 i_0 - 1) , \quad (4.21)$$

and I choose $\phi_1 \sim \pi/4$. With this result, equation (4.8) implies that

$$g_{+,1} g_{-,1} = \cos^2 i_0 . \quad (4.22)$$

Inserting equation (4.22) into equation (4.17) and comparing with equation (3.5), we see that to first order in e^2, the distance determination is independent of eccentricity.

4.3. *Second Order*

Substituting the first-order parameters into equation (4.8) yields

$$g_{+,2}\,g_{-,2} = \left(1 + \frac{1 + \csc^2 i_0}{4}\,\epsilon^2\right)\cos^2 i_0 \,. \quad (4.23)$$

This implies that to second order, the distance is reduced by a fraction $0.40e^4$:

$$D_{\rm SN} = 2D_\times \sqrt{\sec i}(1 - 0.40e^4)\,. \quad (4.24)$$

4.4. *Geometrical Interpretation*

There are two observational constraints on the geometry of the ellipse: the apparent axis ratio and the timing ratio. Suppose that these are consistent with a circular ring at inclination i. Any ellipse that lay at this inclination and had its minor axis aligned with the inclination would have a larger apparent axis ratio than the observed one. If the major axis of the ellipse were aligned with the inclination, the apparent axis ratio would be smaller. Hence, there will always be some intermediate position angle where the ellipse has the same apparent axis ratio as a circle with the same inclination. The above perturbative analysis tells us that this occurs when the position angle is $\pi/4 + O(e^2)$, that is, halfway between the axes. For an ellipse in this position, the timing ratio is very similar to the circular case, exactly the same to first order in e^2. The reason is that while the positions with extreme delay times no longer lie along a diameter, the change in the light-travel times from the diametric case is second order in the angular displacement and hence $O(e^4)$. For an ellipse with position angle $\phi = \pi/4$, the relevant linear dimension for light travel is $\sqrt{a'b'}\sec i$. Since the physical scale of the ring is judged from the deprojected area $\pi D_{\rm SN}^2\,\theta_+\,\theta_-\sec i$, one infers essentially the same distance for an elliptical ring as one would for a circular ring. As the eccentricity becomes large, however, two effects come into play. First, the light path deviates considerably from a straight line across the ellipse. The observed time delays are therefore longer than would be for the case when the two extreme trajectories lay along a single straight line. One therefore overestimates the distance to the supernova if one assumes a circular ring (for which the light paths do lie along a single straight line). Second, the position angle that reproduces the observed projected geometry lies slightly closer to the major axis than to the minor axis. Hence, the projected diameter which most closely approximates the extreme light trajectories is slightly larger than $2\sqrt{a'b'}$. The light-travel time is therefore longer than it would be for a circular ring having the same apparent size. This also causes one to overestimate the distance.

4.5. *Numerical Solution*

It is straightforward to solve equation (4.10) numerically. One may then find the predicted values for η_t and η_θ for any set of ellipse parameters i, ϕ, and e. The likelihood of the data given the model can be evaluated by assuming Gaussian measurement errors. In principle, one should multiply this probability by the prior probability of the set of ellipse parameters and sum over the whole of parameter space. The prior probability of the orientation parameters i and ϕ is well determined: we have no prior knowledge of how the ellipse is oriented, so the prior distribution is uniform in sin i and in ϕ. By contrast, there is no generally agreed upon prior probability for the eccentricity. For example, Panagia et al. believe that it is "physically very hard" to produce eccentricity. I argued that the ring might be eccentric. I avoid this controversy by summing over the orientation angles but reporting the differential distribution in eccentricity.

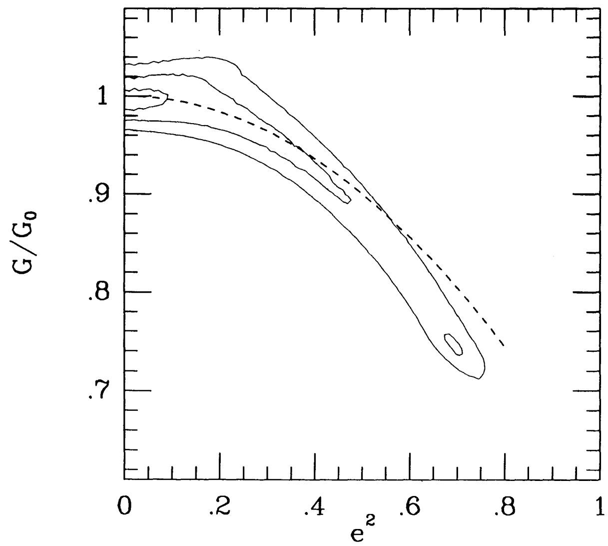

FIG. 1.—Likelihood contours as a function of e^2 and G/G_0, where e is the eccentricity, $G(i, \phi, e)$ is a parameter which enters the determination of $D_{\rm LMC}$, and $G_0 = 2.33$ is the value of G for a circular ring ($e = 0$). Likelihoods are averaged uniformly over the angular variables sin i and ϕ. The contours $m = 1$, 2, and 3 are shown as *solid* curves, where $\exp(-m^2/2) = L/L_{\rm max}$ and L is the likelihood. The second-order perturbation result, $G/G_0 = 1 - 0.40e^4$ is shown as a *dashed* curve. Note that for fixed e^2, the uncertainty in G is small compared to the uncertainty of $D_\times$, the other quantity that enters $D_{\rm LMC}$.

Figure 1 shows likelihood contours for G/G_0 versus e^2, where $G_0 = 2.33$ is the best estimate of G for the circular case ($e^2 = 0$). The contours form a sharply peaked ridge. For a given eccentricity, the measurement uncertainty of G is small compared to the uncertainty of $D_\times$, and it can therefore be ignored. However, the systematic uncertainty due to the possible eccentricity of the ring may be important. To find the most likely value of G, one must estimate the prior probabilities of e^2 and sum over this variable. If one believes that the probability of $e^2 \gtrsim 0.4$ is extremely low, then $G = G_0$. However, even if one relaxes all prior constraints on e^2, one still obtains a hard upper limit,

$$G < G_0 = 2.33\,. \quad (4.25)$$

Substituting this equation into equation (4.16), I find an upper limit on $D_{\rm SN}$ and therefore an upper limit on $D_{\rm LMC}$ of

$$D_{\rm LMC} < 53.2 \pm 2.6 \text{ kpc}\,. \quad (4.26)$$

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APPENDIX A

THE ELLIPSE IN PROJECTION

Suppose that an ellipse has semimajor and semiminor axes a and b. Let ϕ be the position angle of the minor axis relative to the y-axis, and let γ specify the angle of a point on the ellipse relative to the y-axis. Then the (x, y) coordinates of the ellipse are given by

$$x = a \sin \gamma \cos \phi + b \cos \gamma \sin \phi ; \quad y = -a \sin \gamma \sin \phi + b \cos \gamma \cos \phi . \tag{A1}$$

Now suppose that the plane of the ellipse is rotated by an angle i about the x-axis. The coordinates of the projected ellipse will be $\tilde{x} = x$, $\tilde{y} = y \cos i$. Hence, the projected radius, $p = (\tilde{x}^2 + \tilde{y}^2)^{1/2}$ as a function of the parameter γ, is given by

$$\rho^2(\gamma) = A - B \cos (2\gamma) + C \sin (2\gamma) , \tag{A2}$$

where

$$\begin{aligned} A &\equiv \tfrac{1}{4}[(a^2 + b^2)(1 + \cos^2 i) + (a^2 - b^2)(1 - \cos^2 i) \cos (2\phi)] = (ab \cos i) f , \\ B &\equiv \tfrac{1}{4}[(a^2 - b^2)(1 + \cos^2 i) + (a^2 + b^2)(1 - \cos^2 i) \cos (2\phi)] , \\ C &\equiv \tfrac{1}{2} ab(1 - \cos^2 i) \sin (2\phi) , \end{aligned} \tag{A3}$$

and where f is given by equation (4.4). Note that

$$B^2 + C^2 = (ab \cos i)^2 (f^2 - 1) . \tag{A4}$$

The major and minor axes of the projected ellipse are defined by the maximum and minimum values of $\rho(\gamma)$. From equation (A2), these are found at values of γ defined by

$$\tan (2\gamma) = -\frac{B}{C} . \tag{A5}$$

The two roots of equation (A2) are therefore

$$\rho_{1,2}^2 = A \pm (B^2 + C^2)^{1/2} . \tag{A6}$$

I designate the projected axes $a' = \rho_1$ and $b' = \rho_2$. Equations (A6) and (A4) then imply that $\pi a' b' = \pi ab \cos i$. From equation (A6), I then find

$$\frac{b'}{a'} = \frac{A - (B^2 + C^2)^{1/2}}{[A^2 - (B^2 + C^2)]^{1/2}} , \tag{A7}$$

which can easily be evaluated using equation (A4) to yield equation (4.3).

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